Chapter 24
The Hayashi Line

We have seen that convection can occur in quite different regions of a star. In this section we consider the limiting case of *fully convective stars,* i.e. stars which are convective in the whole interior from centre to photosphere, while only the atmosphere remains radiative.

The Hayashi line (HL) is defined as *the locus in the Hertzsprung–Russell diagram of fully convective stars of given parameters* (mass M and chemical composition). Note that for each set of the parameters, such as mass or chemical composition, there is a separate Hayashi line. These lines are located far to the right in the Hertzsprung–Russell diagram, typically at $T_{\text{eff}} \approx 3,000 \ldots 5,000\,\text{K}$, and they are very steep, in large parts almost vertical.

From the foregoing definition one may not immediately realize the importance of this line. However, the HL also represents *a borderline between an "allowed" region* (on its left) *and a "forbidden" region* (on its right) in the Hertzsprung–Russell diagram for all stars with these parameters, provided that they are in hydrostatic equilibrium and have a fully adjusted convection. The latter means that, at any time, the convective elements have the properties (for instance the average velocity) required by the mixing-length theory. Changes in time of the large-scale quantities of the stars are supposed to be slow enough for the convection to have time to adjust to the new situation; otherwise one would have to use a theory of time-dependent convection. Since hydrostatic and convective adjustment are very rapid, stars could survive on the right-hand side of the HL only for a very short time.

In addition, parts of the early evolutionary tracks of certain stars may come close to, or even coincide with, the HL. It is certainly significant for the later evolution of stars, which is clearly reflected by observed features (e.g. the ascending branches of the Hertzsprung–Russell diagrams of globular clusters). One may even say that the importance of the HL is only surpassed by that of the main sequence. It is all the more surprising that its role was not recognized until the early 1960s when the work of Hayashi (1961) appeared. The late recognition of the HL may partly be because its properties are derived from involved numerical calculations. In the following we will use extreme simplifications in order to make some basic characteristics of the HL plausible.

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24.1 Luminosity of Fully Convective Models

Let us consider the different ways in which the luminosity is coupled to the pressure-temperature stratification of radiative and convective stars.

For regions with radiative transport of energy, we can write the "radiative luminosity" $l_{\rm rad} = 4\pi r^2 F_{\rm rad}$ according to (7.2) as

$$l_{\rm rad} = k'_{\rm rad} \nabla, \tag{24.1}$$

with the usual notation $\nabla = d \ln T / d \ln P$ and the "radiative coefficient of conductivity"

$$k'_{\rm rad} = \frac{16\pi acG}{3} \frac{T^4 m}{\kappa P}. \tag{24.2}$$

If a stratification of P and T is given, then the luminosity l_{rad} is obviously determined and can be easily calculated from (24.1).

For convective transport of energy by adiabatically rising elements we can write accordingly from (7.7) the convective luminosity as

$$l_{\rm con} = k'_{\rm con} (\nabla - \nabla_{\rm ad})^{3/2} \tag{24.3}$$

with the coefficient

$$k'_{\rm con} \frac{\pi}{\sqrt{2}} \left(\frac{\ell_m}{H_P} \right)^2 r^2 c_P T (\varrho P \delta)^{1/2}. \tag{24.4}$$

Here we have made use of the hydrostatic equation and the definition (6.8) of the pressure scale height. The mixing length $\ell_{\rm m}$ was defined in Sect. 7.1.

In principle, we can again assume the luminosity to be determined using (24.3) for a given P–T stratification. In practice, however, we would never be able to calculate $l_{\rm con}$ from this equation for the stellar interior, since it would require the knowledge of the value of ∇ with inaccessible accuracy. The point is that $l_{\rm con}$ is not proportional to the gradient ∇ itself but rather to a power of the excess over the adiabatic gradient, $\nabla - \nabla_{\rm ad}$, which may be as small as 10^{-7} for very effective convection (see Sect. 7.3). Therefore the convective conductivity $k'_{\rm con}$ must be very high, since large luminosities $l_{\rm con}$ are carried. This may be looked at in another way: by solving (24.3) for ∇ and writing

$$\nabla = \nabla_{\rm ad}(1 + \varphi), \tag{24.5}$$

we see that the luminosity influences the T gradient only through the tiny correction $\varphi (\approx 10^{-7})$:

$$\varphi = \left[\frac{l_{\rm con}}{\nabla_{\rm ad}^{3/2} k'_{\rm con}} \right]^{2/3}. \tag{24.6}$$

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ıple Description of the Hayashi Line

ive some typical properties of the HL analytically, we shall use an
e model for fully convective stars (Further refinements of the picture,
e, would not be worth the large additional complications involved.).
een that nearly all of the interior part of convective stars has an
ification, such that $d \ln T / d \ln P = \nabla_{ad}$. We shall assume that
lation between P and T holds for the whole interior up to the
.e. we neglect the superadiabaticity in the range immediately below
re. We also neglect the depression of ∇_{ad} in those regions near the
e H and He are partially ionized (see Figs. 11.2 and 14.1). We thus
e ∇_{ad} to be constant throughout the star's interior, say $\nabla_{ad} = 0.4$,
value for a fully ionized ideal gas. With these simplifications we
oduce errors in the $P-T$ stratification. However, they will be nearly
neighbouring models, and we can hope to obtain at least the correct
ehaviour.

ave for the whole interior the simple $P-T$ relation

$$P = C T^{1+n} , \qquad (24.8)$$

is polytropic with an index $n = 1/\nabla_{ad} - 1 = 3/2$, and we can use
results for such stars (see Chap. 19). The constant C is related to the
constant K defined in (19.3). With $P = \Re \varrho T / \mu$, one finds $C =$
$^{1+n}$. K and C are constant only within one model, but vary from star

to star, which means that we do not have a mass-radius relation. From (19.9) and (19.19) it follows that

$$K \sim \varrho_c^{1/3} A^{-2} \sim \varrho_c^{1/3} R^2 \sim M^{1/3} R, \tag{24.9}$$

so that

$$C = C' R^{-3/2} M^{-1/2}, \tag{24.10}$$

where the constant C' is known for given n and μ.

Relation (24.8) is now assumed to hold as far as the photosphere, where the optical depth $\tau = 2/3$, $P = P_0$, $T = T_{\mathrm{eff}}$, $r = R$, and $m = M$. Above this point we suppose to have a radiative atmosphere with a simple absorption law of the form

$$\kappa = \kappa_0 P^a T^b. \tag{24.11}$$

Integration of the hydrostatic equation through the atmosphere yields the photospheric pressure [cf. (11.13), where $\bar{\kappa}$ is replaced by (24.11)] as

$$P_0 = \text{constant} \left(\frac{M}{R^2} T_{\mathrm{eff}}^{-b} \right)^{\frac{1}{a+1}}. \tag{24.12}$$

We now fit this to the interior solution by setting $P = P_0, T = T_{\mathrm{eff}}$ in (24.8) and then eliminating P_0 with (24.12). For given values of M and μ this yields a relation between R and T_{eff}, or between R and L, since $L \sim R^2 T_{\mathrm{eff}}^4$. Thus, any value of R corresponds to a certain point in the Hertzsprung–Russell diagram. The interior solutions form a one-dimensional manifold, since the constant C contains the free parameter R for given M [and given μ, see (24.10)]. In the Hertzsprung–Russell diagram this is reflected by a one-dimensional manifold of points defining the Hayashi line.

The fitting procedure is illustrated in Fig. 24.1. Each interior solution of the form (24.8) with $n = 3/2$ is represented in this diagram by a straight line:

$$\lg T = 0.4 \lg P + 0.4 \left(\frac{3}{2} \lg R + \frac{1}{2} \lg M - \lg C' \right). \tag{24.13}$$

For fixed values of M and μ, each of these lines is characterized by a value of R. The atmospheric solutions (24.12) are another set of straight lines in Fig. 24.1:

$$(a+1) \lg P_0 = \lg M - 2 \lg R - b \lg T_{\mathrm{eff}} + \text{constant}. \tag{24.14}$$

The intersection of a line of the first set with a line of the second set, both with the same value of R, fixes the corresponding value of T_{eff} (and of P_0). From R and T_{eff} we have L, i.e. a point in the Hertzsprung–Russell diagram. We then obtain the Hayashi line by a continuous variation of R.

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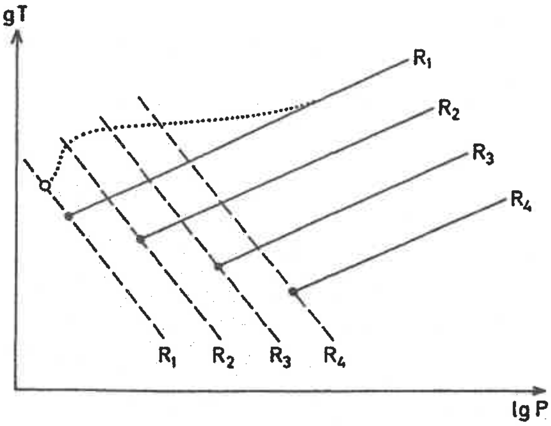


Fig. 24.1 Fit of a polytropic ($n = 3/2$) interior solution (*solid line*) with an atmospheric condition (*dashed line*) for different values of R ($R_1 > R_2 > R_3 > R_4$). The photospheric points obtained by this fit are marked by *dots*. The *dotted line* illustrates schematically the effects of superadiabatic convection and depression of $\nabla_{\rm ad}$ in an ionization zone for $R = R_1$

The formalism for this procedure, as described, yields immediately an equation for the Hayashi line in the Hertzsprung–Russell diagram:

$$\lg T_{\rm eff} = A \lg L + B \lg M + \text{constant} \tag{24.15}$$

with the coefficients

$$A = \frac{0.75a - 0.25}{b + 5.5a + 1.5}, \quad B = \frac{0.5a + 1.5}{b + 5.5a + 1.5}. \tag{24.16}$$

We now need typical values for the exponents a and b in the atmospheric absorption law (24.11). An important property of fully convective stars can immediately be concluded from the discussion in Sect. 11.3: such stars must have very low values of $T_{\rm eff}$, i.e. *the Hayashi line must be far to the right in the Hertzsprung–Russell diagram.* For atmospheres this means that in most parts $T \lesssim 5 \times 10^3$ K, and H^- absorption will provide the dominant contribution to κ. If hydrogen is essentially neutral, the free electrons necessary for the formation of H^- ions are provided by the heavier elements (see Sect. 17.5). A very rough interpolation gives $a \simeq 1, b \simeq 3$. With these values (24.16) yields the coefficients

$$A = 0.05, \quad B = 0.2. \tag{24.17}$$

According to (24.15), the slope of the Hayashi line in the Hertzsprung–Russell diagram is $\partial \lg L / \partial \lg T_{\rm eff} = 1/A$. Since $A \ll 1$, we conclude that *the Hayashi line must be very steep.* The value of $B \equiv \partial \lg T_{\rm eff} / \partial \lg M$ means that *the Hayashi line shifts slightly to the left in the Hertzsprung–Russell diagram for increasing M.* These qualitative predictions, although derived from very crude assumptions, are fully supported by the numerical results (see Fig. 24.3).

Let us consider once more the reason for the steepness of the HL. At the photosphere the pressures P_{0i} of the interior solution (24.8), (24.10) and P_{0a} of the atmospheric solution (24.12) vary for constant M as

$$P_{0i} \sim \frac{T_{\rm eff}^{2.5}}{R^{3/2}}, \quad P_{0a} \sim \frac{T_{\rm eff}^{-\frac{b}{a+1}}}{R^{\frac{2}{a+1}}}. \tag{24.18}$$

First of all, we expect a very steep HL for small positive values of a. In fact, for $a = 1/3$, P_{0i} and P_{0a} have the same dependence on R; then $T_{\rm eff}$ does not vary with R (and L), and the line is vertical. If this is not quite fulfilled, the fit $P_{0i} = P_{0a}$ requires the smaller variations of $T_{\rm eff}$ with varying R, the more different the two exponents of $T_{\rm eff}$ in (24.18) are, i.e. the larger b.

The basic approximations made were to neglect the depression of $\nabla_{\rm ad}$ in ionization zones and to ignore superadiabatic convection. The dotted line in Fig. 24.1 indicates how these effects change the $P-T$ structure relative to a simple polytrope. One sees that they tend to increase the effective temperature. The precise value of $T_{\rm eff}$ obviously depends on the detailed structure of the outermost envelope. The extension and the depth of the ionization zones and the superadiabatic layers change systematically with L. This has the consequence that, in better approximations, the coefficient A in (24.15) changes sign at $L \simeq L_\odot$. It is positive for smaller L, and negative for larger L, so that the HL is convex relative to the main sequence.

Another important conclusion is that the whole uncertainty which remained in the mixing-length theory of ineffective convection must occur as a corresponding uncertainty in the precise value of $T_{\rm eff}$ for the HL.

Finally, we note that the chemical composition enters into the position of the HL in two ways. The interior is affected, since the polytropic constant C depends on μ via C' [see (24.10)], and the outer layers are particularly affected via the opacity κ.

24.3 The Neighbourhood of the Hayashi Line and the Forbidden Region

We now consider stars in hydrostatic equilibrium that are close to, but not exactly on, their HL. Certainly the stars cannot be fully convective with an adiabatic interior (otherwise they would be on the HL). Their interior is then no longer a simple polytrope. They do not even have to be chemically homogeneous, since they are not fully mixed by the turbulent motions. We must therefore expect that an analytical treatment will be much more complicated. We will nevertheless try to give some simple arguments which may help to make the numerical results plausible. In the following, we treat models with a fixed value of M and the same chemical composition (at least in their outer layers).

An important indication can be obtained from the discussion of the envelope integrations in Sect. 11.3. When integrating inwards into models with different $T_{\rm eff}$ (but with the same parameters M and μ and, say, the same L), we will reach a radiative region the earlier, the larger $T_{\rm eff}$. In other words, in models left of the HL we will encounter a radiative region before reaching the centre. In these regions, the gradient $\nabla < \nabla_{\rm ad}$. Let us consider some average $\bar{\nabla}$ obtained by averaging over the whole interior (where we again neglect the complications in the outermost parts of the envelope). On the HL we have $\bar{\nabla} = \nabla_{\rm ad}$. In a model to the left of the HL the radiative part decreases the average value such that $\bar{\nabla} < \nabla_{\rm ad}$. This suggests that we would have to allow $\bar{\nabla} < \nabla_{\rm ad}$ in models to the right of the HL.

In order to prove this we treat models with a constant gradient $\nabla = \bar{\nabla}$ in the interior and vary $\bar{\nabla}$ slightly around $\nabla_{\rm ad}$. We then have again polytropic stars with slightly different n (around 3/2). The interior solution is written as

$$P = C_n T^{1+n}, \tag{24.19}$$

where $\bar{\nabla} = (1+n)^{-1}$ and, similarly to (24.10),

$$C_n = C_n' \mu^{-n-1} M^{1-n} R^{n-3}. \tag{24.20}$$

From now on we measure R and M in solar units. Then

$$C_n' = \frac{\Re^{n+1}}{4\pi G^n}(n+1)^n \left[-\left(\frac{dw}{dz}\right)_{z=z_n} \right]^{n-1} z_n^{n+1} R_\odot^{n-3} M_\odot^{1-n}. \tag{24.21}$$

We extend relation (24.19) to the photosphere ($P = P_0, T = T_{\rm eff}$), where we again eliminate P_0 by (24.12) and R by the relation $R = c_2 L^{1/2} T_{\rm eff}^{-2}$. This gives the locus in the Hertzsprung–Russell diagram. The factor of proportionality in (24.12) may be called c_1. Choosing for simplicity $a = 1, b = 3$ in the opacity law, we obtain

$$\lg T_{\rm eff} = \alpha_1 \lg L + \alpha_2 \lg M + \alpha_3 \lg \mu + \alpha_4 \lg C_n' + \alpha_5 \lg c_1 + \alpha_6 \lg c_2, \tag{24.22}$$

where the coefficients depend on n:

$$\alpha_1 = \frac{2-n}{13-2n}, \quad \alpha_2 = \frac{2n-1}{13-2n}, \quad \alpha_3 = \frac{2(1+n)}{13-2n},$$
$$\alpha_4 = \frac{-2}{13-2n}, \quad \alpha_5 = -\alpha_4, \quad \alpha_6 = 2\alpha_1. \tag{24.23}$$

The α_i do not vary too much with small deviations of n from 3/2. This means, for example, since α_1 determines the slope, lines of neighbouring values of n are nearly parallel to the HL. Without loss of generality, we may consider particular models on and close to the HL with $L = M = \mu = 1$. The variation of $\lg T_{\rm eff}$ with n is then only due to the variation of the last three terms in (24.22). One finds that

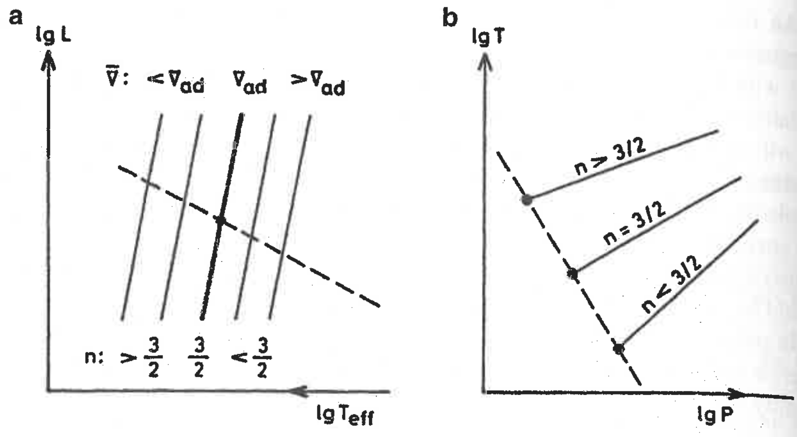


Fig. 24.2 (**a**) In the Hertzsprung–Russell diagram, the Hayashi line ($n = 3/2$, *heavy line*) is indicated, together with some neighbouring lines for interior polytropes with $n > 3/2$ and $< 3/2$. (**b**) The same as Fig. 24.1 but with three different polytropic interior solutions for the same value of R

$\partial \lg T_{\text{eff}}/\partial n > 0$: the stars move to the right in the Hertzsprung–Russell diagram with decreasing n (i.e. increasing $\bar{\nabla}$).

Thus, we have to expect the following situation (see Fig. 24.2): left of the HL we have $\bar{\nabla} < \nabla_{\text{ad}}$ and some part of the model is radiative. On the HL, the model is fully convective with $\bar{\nabla} = \nabla_{\text{ad}}$. Models to the right of the HL should have $\bar{\nabla} > \nabla_{\text{ad}}$, which means that they should have a superadiabatic stratification in their very interior (aside from the outermost zone of ineffective convection).

The mixing-length theory has shown that a negligibly small excess of ∇ over ∇_{ad} suffices in order to transport any reasonable luminosity in the deep interior of stars. Then, what happens with a star that by some arbitrary means (e.g. initial conditions) has been brought to a place to the right of the HL, such that some region in its deep interior has remarkably large values of $\nabla - \nabla_{\text{ad}} > 0$? The results are large convective velocities $v_{\text{conv}} \sim (\nabla - \nabla_{\text{ad}})^{1/2}$ and corresponding convective fluxes [cf. (24.3)]. These cool the interior and heat the upper layers rapidly until the gradient is lowered to $\nabla \approx \nabla_{\text{ad}}$ and the star has moved to the HL. This will happen within the short timescale for the adjustment of convection.

Another possibility for a star being situated to the right of its HL is, of course, that it is not in hydrostatic equilibrium (which is assumed for the interior solution). But a deviation from this equilibrium will be removed in the timescale for hydrostatic adjustment, which is even shorter.

Therefore the HL is in fact a borderline between an "allowed" region (left) and a "forbidden" region (right) for stars of given M and composition that are in hydrostatic equilibrium and have a fully adjusted convection.

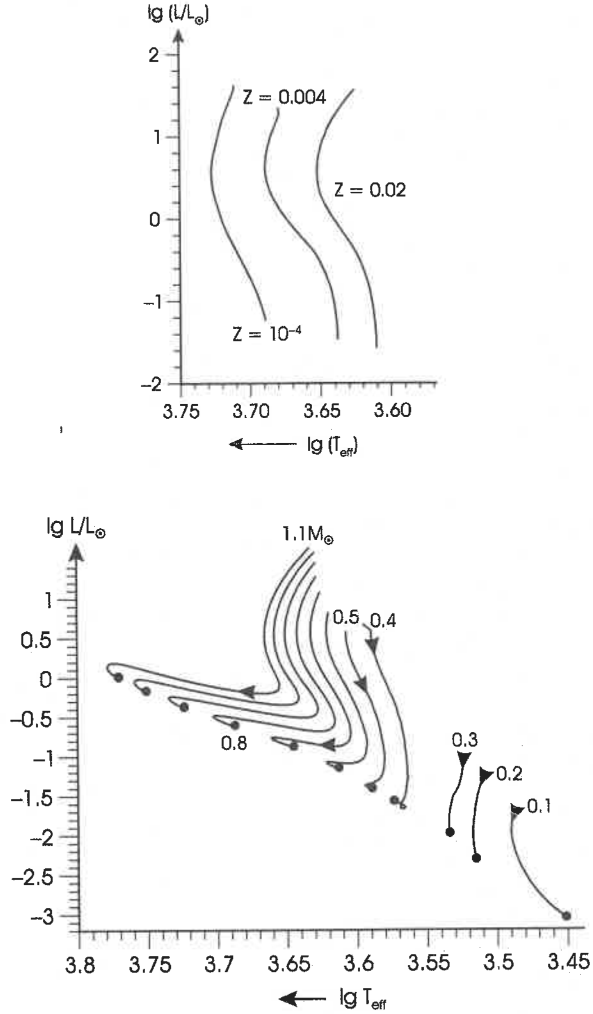


Fig. 24.3 *Top*: The position of Hayashi lines for stars of $M = 0.8M_\odot$ but different composition. The helium content is always 0.245, while Z varies from 10^{-4} to 0.02. *Bottom*: Pre-main-sequence evolution along the Hayashi line to the zero-age main sequence for stars between 0.1 and 1.1 $M_\odot$ and a solar-like composition (Data courtesy S. Cassisi)

24.4 Numerical Results

There are many results available giving the position of Hayashi lines for stars of widely ranging mass and chemical composition and for different assumptions in the convection theory. The latter concerns in particular the ratio of mixing length to pressure scale height used for calculating the superadiabatic envelope.

Figure 24.3 shows typical results of calculations for stellar masses of up to 1.1 $M_\odot$. One sees that indeed the HLs plotted here are very steep, the exact slope depending mainly on L. The dependence on M (lower panel) is roughly given by

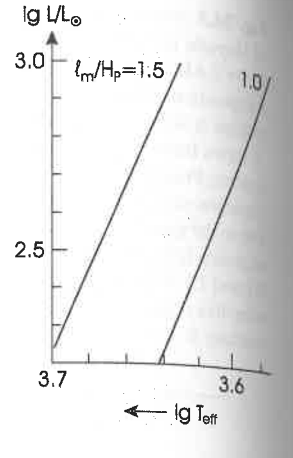


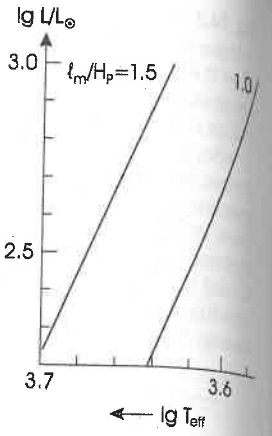
Fig. 24.4 The Hayashi line for $M = 5M_\odot$ with two different assumptions for the ratio of mixing length to pressure scale height (After Henyey et al. 1965)

$\partial \lg T_{\text{eff}}/\partial \lg M \approx 0.1$, i.e. we find the expected weak increase of T_{eff} with M [cf. (24.22)]. The dependence on chemical composition (top panel) is, however, very different from that given by (24.23), which yields $\alpha_3 = 0.5$. It predicts only a slight decrease in T_{eff}, when increasing the metallicity from 10^{-4} to 0.02, as in the left panel of the figure. In that case $\lg \mu$ changes from -0.229 to -0.226, and $\lg T_{\text{eff}}$ should increase by ≈ 0.002. The numerical result instead is $\partial \lg T_{\text{eff}}/\partial \lg \mu \approx -26$, i.e. with increasing molecular weight T_{eff} is strongly reduced!

As mentioned earlier the chemical composition enters in several ways. A very important factor certainly is the opacity in the atmosphere. For $T_{\text{eff}} \lesssim 5{,}000$ K the dominant absorption is due to H^-, and κ then is proportional to the electron pressure, which in turn is proportional to the abundance of the easily ionized metals. It turns out that a *decrease* of their abundance (usually comprised in Z) by a factor 10 shifts the HL by $\Delta \lg T_{\text{eff}} \approx +0.05$ to the left in the Hertzsprung–Russell diagram. This explains the large effect of changing the composition seen in Fig. 24.3. However, Fig. 24.4 shows that roughly the same shift can be obtained by the comparatively small increase of l_m/H_P from 1 to 1.5. The uncertainty of the convection theory, therefore, severely limits our knowledge of the HL.

The typical S-shape of the numerical Hayashi tracks in Fig. 24.3 are the result of the sign change of coefficient A in (24.15), which was mentioned at the end of Sect. 24.2. At the lowest end of the Hayashi tracks the models develop a radiative core and begin to bend back to the main sequence, where they end once nuclear burning has started at the centre, supplying the energy radiated from the surface. This is the situation discussed in Sect. 24.3.

Thus, the HLs are far away from the main sequence in the upper part of the diagram, and approach it in the lower part. This fact will turn out to influence the evolutionary tracks of stars of different M. Recall that the main-sequence stars were found to be fully convective for $M \lesssim 0.25M_\odot$ (see Sect. 22.3). This obviously means that the corresponding Hayashi lines cross the main sequence there.



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24.5 Limitations for Fully Convective Models

In order to describe the HL, we have considered models for which the convection was postulated to reach from centre to surface. This provided a polytropic interior structure with typical decoupling from the luminosity. We have not yet asked whether the physical situation will in fact allow the onset of convection throughout the star. This depends on the distribution of the energy sources.

According to the Schwarzschild criterion (6.13), a chemically homogeneous layer will be convective if

$$\nabla_{rad} \geq \nabla_{ad}, \tag{24.24}$$

where the radiative gradient [see (5.28)] is

$$\nabla_{rad} \sim \frac{\kappa l P}{T^4 m}. \tag{24.25}$$

If the energy sources were completely arbitrary, we could choose their distribution so that (24.24) is violated at some point and the model could not be *fully* convective. A trivial example would be a central core without any sources, with the result that there $l = 0$, i.e $\nabla_{rad} = 0$. Then the core must be radiative. On the other hand, we have the best chance of finding convection throughout a star of given L if the sources are highly concentrated towards the centre (in the extreme: a point source), which gives almost $l = L$ everywhere.

We consider a contracting polytrope (see Sect. 20.3) without nuclear energy sources, which is of interest for early stellar evolution. According to (20.41) the energy generation rate is then only proportional to T, which means a rather weak central concentration. For the sake of simplicity we even go a step further and assume constant energy sources with

$$\frac{l}{m} = \frac{L}{M} = \text{constant}. \tag{24.26}$$

We again use the opacity law (24.11) and the polytropic relation (24.8) with $n = 1.5$ (corresponding to $\nabla = \nabla_{ad} = 0.4$). Equation (24.25) then gives

$$\nabla_{rad} \sim \frac{L}{M} C^{1+a} T^{b-4+2.5(1+a)}. \tag{24.27}$$

For a typical Kramers opacity with $a = 1, b = -4.5$ this becomes $\nabla_{rad} \sim T^{-3.5}$. Indeed, for all reasonable interior opacities, ∇_{rad} has a minimum at the centre and increases outwards. Therefore the centre is the first point in a fully convective star where ∇_{rad} drops below ∇_{ad} (and a radiative region starts to develop) if L decreases below a minimum value L_{min}.

The constant C depends on M and R as given by (24.10), and $T \sim T_c \sim M/R$ after (20.24). Introducing this into (24.27) we obtain

$$\nabla_{\rm rad} \sim L M^{b-5+2(1+a)} R^{-b+4-4(1+a)}. \tag{24.28}$$

Let us again set $a = 1$, $b = -4.5$, which gives

$$\nabla_{\rm rad} \sim L M^{-5.5} R^{0.5}. \tag{24.29}$$

For models on the HL, the effective temperatures vary only a very little and we simply take $R \sim L^{1/2}$. Then,

$$\nabla_{\rm rad} \sim L^{1.25} M^{-5.5}. \tag{24.30}$$

For any given value of M the luminosity reaches $L_{\rm min}$ if the central value of $\nabla_{\rm rad}$ has dropped to 0.4. According to (24.30), $L_{\rm min}$ depends on M as

$$L_{\rm min} \sim M^{4.4}. \tag{24.31}$$

This minimum luminosity (down to which models of the specified type on the HL remain fully convective) decreases strongly with M. The decrease is in fact steeper than that given by the $M - L$ relation of the main sequence. This provides the possibility that the HL for very small M can cross the main sequence without reaching $L_{\rm min}$.

Note, however, that strictly speaking a "minimum luminosity" always refers to a fixed distribution of the energy sources.