17.5 *Nuclear Reactions*

We've now seen how Fig. 36 can be populated with tracks representing the central conditions for a range of stars. We can now also populate the T-ρ diagram with a set of orthogonal curves describing nuclear energy production in the cores of our stars.

What is a useful characteristic energy production rate ϵ to use for these tracks? We know that in the absence of fusion, stars can be (briefly) heated via gravitational (Kelvin-Helmholtz) contraction. We'll therefore require that nuclear burning produces enough energy to overcome that contraction. In other words,

$$(420)\quad \tau_{KH} \sim \frac{GM^2/R}{L} \sim \frac{u}{\epsilon_V}$$

where

$$(421)\quad u = \frac{3}{2}nkT$$

and the volumetric and mass-based energy rates are related as

$$(422)\quad \epsilon_V = \epsilon_m \rho.$$

This implies that the relevant energy production rate of for any given fusion chain, and over a wide range of stars, is approximately

$$(423)\quad \epsilon_{m,0} \sim 10 \text{ erg g}^{-1} \text{ s}^{-1}.$$

We saw in Sec. 14.5 that the an approximate, general form of ϵ (Eq. 354) is

$$\epsilon = \epsilon_0 \rho^\lambda T^\nu.$$

Thus at the threshold $\epsilon_{m,0}$, we have

$$(424)\quad \log \rho = -\frac{\nu}{\lambda} \log T + \frac{1}{\lambda} \log \left(\frac{\epsilon_{m,0}}{\epsilon_0} \right).$$

Each reaction has its own corresponding coefficient. Successive stages of thermonuclear fusion turn on at higher temperatures and have steeper dependences on T in particular (with $\lambda \gg 1$ only in the rarely-approached pyconuclear regime).

The features of these curves in Fig. 36 can then be described as follows:

- **p-p chain**: At high ρ, ignites at fairly low T. $\nu \approx 4$.
- **CNO cycle**: Dominates at sufficiently high T and low ρ. Thus given the right raw materials, CNO can actually ignite first as a cloud condenses to form a star. Still just converting protons into He, so continuous with the pp track, but now $\nu \approx 16$.
- 3α: Ignites at $\sim 10^8$ K, with a very steep T dependence: $\nu \approx 40$.

- **C+C**: Ignites at $\sim 6 \times 10^8$ K, with an even steeper (practically vertical) slope.
- **O+O**: Ignites at $\sim 10^9$ K, with an even steeper (practically vertical) slope.
- **Si**: Ignites at $\sim 2 \times 10^9$ K, with an even steeper (practically vertical) slope.

17.6 *Stability*

For any reasonable duration, dynamical stability will confine our stars to certain regions of the T-ρ diagram. In particular, our stars will become unstable whenever γ approaches 4/3. Thus the "ideal gas" and "non-relativistic degenerate" zones are fair game, but in both the radiation-dominated regime and in the ultra-relativistic degenerate regime the conditions may verge perilously close to instability.

Other interesting instabilities can also develop. At the highest temperatures $T \gtrsim 10^9$ K, radiation pressure is often the dominant support. But the **pair instability** can remove or decrease that radiation support, leading to collapse. Instead of providing support,

(425) $\gamma \rightarrow e^+ + e^-$

which means

(426) $kT \approx 2 m_e c^2$.

Since 1 eV$\sim$12,000 K and m_e=0.5 MeV, this should set in at around

(427) $T_{pp} \approx 10^{10}$ K.

In practice, pair instability sets in considerably earlier because the high-energy tail of the photon distribution can begin pair production long before the kT bulk of the photons reach that level, so it can be relevant for $T \gtrsim 5 \times 10^8$ K.

One final realm of instability is caused by **photodissociation of nuclei**. When $T \gtrsim 3 \times 10^9$ K, individual photons have enough energy to return all the lost binding energy back into heavy nuclei. The most important example is

(428) $\gamma +^{56}\mathrm{Fe} \rightarrow 14^4\mathrm{He}$,

which plays an important role in some supernovae.

17.7 *A schematic overview of stellar evolution*

We're finally in a position to piece together a basic-level astrophysical understanding of the evolution of a star. How do the central conditions of different objects evolve on the T-ρ diagram (Fig. 36).

- **One mass, one fate.** Each particular mass of star follows a distinct track, as described in Sec. 16.5.

- **Start low, end high.** We haven't talked much about the earliest stages of star formation, but we know space is big, empty, and cold. So any star presumably begins the earliest stages of its life at the relatively low temperatures and densities of the interstellar medium.
- **Move along home.** As a gas cloud approaches becoming a *bona fide* star, it contracts and radiates on the Kelvin-Helmholtz timescale (Sec. 10.5). As it contracts, no mass is lost so ρ must increase. And by the Virial Theorem (Eq. 219), T must increase as well.
- **Stop! in the name of fusion.** Eventually the core conditions will hit one of our fusion tracks. We defined our energy production tracks in Sec. 17.5 such that nuclear luminosity balanced the luminosity of gravitational collapse. So the star will remain $\sim$stable at this point for τ_{nuc}.
- **Get up again.** Once nuclear fuel is exhausted in the core, to maintain stability contraction must resume. ρ_c and T_c begin to increase again.
- **Rinse and repeat.** Pause at each nuclear burning threshold, for ever-briefer periods of time, until either a degenerate zone or unstable zone is reached.
- **Just fade away.** Once a star enters the non-relativistic degenerate zone, it's game over. Once any residual fusion is completed, the star can no longer contract to heat and support itself. It will just sit at constant ρ_c, gradually cooling and fading away: it is now a white dwarf. This is the fate of all stars with $M_* \lesssim 1.4 M_\odot$, the **Chandrasekhar Mass**.
- **Do not burn.** Even lowest-mass "stars" will contract and evolve up and to the right in T-ρ space, but for $M \lesssim 0.08 M_\odot$ the track will never intersect the pp-chain burning track. Thus they will become degenerate before ever undergoing fusion; these are **brown dwarfs**.
- **Your star is so massive...** The most massive stars will follow tracks along the upper border of the radiation-dominated regime. This border has the same slope as our equation-of-state tracks, implying that there is some maximum mass that stars can have – any more massive and they would reach $\gamma = 4/3$ and become entirely unstable.
- **Do not pass Go.** Fig. 36 also reveals the final, often-fatal fates of various stars. This includes:
  - Lowest-mass stars: these burn H$\rightarrow$He for many Gyr, then become degenerate **Helium white dwarfs.**
  - Stars $< 1.4 M_\odot \approx M_{Ch}$ produce He and then later also undergo the 3α process. They spend the rest of their days as **carbon/oxygen white dwarfs.**
  - These white dwarfs will occasionally evolve across an ignition line while inside the degenerate region. In this case, we have a **nuclear runaway** and the star will fuse all available fuel almost instantly

(on a thermal conduction timescale, just a few seconds). The best example is the **helium flash**, which occurs for stars at or just below $1 M_\odot$. The 3α line is reached right near the non-relativistic degenerate zone, and the core luminosity will spike as high as $10^{11} L_\odot$ for a few seconds. This intense burst only slowly "leaks out" into observable regions, but it quickly melts away the core degeneracy.

– Stars $> M_{Ch}$ will succumb to the instabilities lurking at high T. Most will pass through multiple levels of fusion burning, all the way up to ^{56}Fe, before finally reaching the photodissociation threshold. They will die as **core-collapse supernovae.** The most massive stars are very rare, but some may end their lives via the pair-production instability instead.

17.8 *Timescales: Part Deux*

Note that our discussion so far has left out an explicit treatment of timescales: how long does a star sit at any given nuclear burning threshold, how long does it take to pass from one threshold to the next, and how long does it take a white dwarf to cool? For now, let's merely realize that any given stage of fusion will continue for roughly τ_{nuc} (Eq. 207) while contraction occurs over roughly τ_{KH} (Eq. 205). So the Sun took only a few 10s of Myr to collapse from a gas cloud into a young zero-age main-sequence star, but it will sit on the H$\rightarrow$He burning threshold for roughly 10^4 Myr. Since nuclear timescales scale roughly as M^{-3}, more massive stars will fuse up all available hydrogen in just a few 10s of Myr — the lowest-mass stars will take trillions ($> 10^6$ Myr!).