22 Black Holes

22.1 *Useful references*

- Kippenhahn, Weiger, and Weiss, 2nd ed., Ch. 39

22.2 *Introduction*

We've almost completed our astrophysical survey of stars, their evolution, and the final end products. Just to recap:

Initial Mass	Fate	Final Mass
$\lesssim 13 M_{\rm Jup}$	Planet	same
$\sim 13 M_{\rm Jup} - \sim 0.08 M_\odot$	Brown dwarf	same
$\lesssim 0.08 M_\odot$	Brown dwarf	same
$0.08 M_\odot - 0.8 M_\odot$	Lives on MS for $> t_{\rm Hubble}$	same
$0.8 M_\odot - \boxed{7 M_\odot}$	White dwarf	$0.6 M_\odot - 1.4 M_\odot$
$\boxed{7 M_\odot} - \boxed{20 M_\odot}$	Neutron star	$1.4 M_\odot - \underline{3 M_\odot\ (?)}$
$\boxed{\gtrsim 20 M_\odot}$	Black hole	$\underline{\gtrsim 3 M_\odot\ (?)}$

In this table, initial masses in $\boxed{\text{boxes}}$ are uncertain due to poorly understood astpects of mass loss during stellar evolution. On the other hand, final masses that are underlined above are uncertain because the equation of state of neutron stars is only poorly known. But at final masses $\gtrsim 3 M_\odot$, no known physics provides a pressure that can hold up a star. The increase in pressure itself is ultimately self-defeating: it gravitates! Eventually the point is reached where support would require infinite pressure; nothing can hold it up. General relativity tells us that it must collapse, leaving a black hole behind.

22.3 *Observations of Black Holes*

Like neutron stars, the concept of black holes was invented before any observational evidence arose. Even 18th-century natural philosophers considered the impact of sufficient gravity on corpuscular light (i.e., photons). Relativity put the discussion on firmer and more accurate footing, but decades passed before the impact of event horizons, rotating black holes, etc. were recognized. In the last half-century observers have steadily built up a catalog of objects that are

- **Massive** — i.e., $> 3 M_\odot$ and so more massive than any plausible neutron star equation-of-state can support;
- **Compact**
- **Dark.**

This catalog includes many objects of masses $M \sim 5 - 25 M_\odot$ (stellar remnants; see Fig. 44), along with objects with $M \sim 10^6 - 10^9 M_\odot$ (**supermassive black holes**) at the centers of our and other galaxies. Evidence for **intermediate-mass black holes** remains inconclusive despite considerable searches.

Many of the first such stellar-mass black holes were discovered as bright X-ray sources. One of the earliest was Cygnus X-1 (i.e., the brightest X-ray source in the constellation Cygnus), over which Steven Hawking lost a bet with Kip Thorne. Another was V404 Cygni (a variable star in the same constellation), identified earlier but which underwent a massive outburst in 2015 – at peak brightness, the system was 50× brighter than the Crab Nebula (supernova remnant) in X-rays. In all these systems, the X-rays arise from hot gas (at millions of K) in an accretion disk spiraling down into the black hole. Most of these systems are binaries, and the accreting material is stripped from a "normal" star (pre-collapse, pre-supernova) by the black hole. Thus the component masses can be measured using the tools discussed in Sec. 4.

For V404 Cyg, the binary mass function (Eq. 7) is

$$(577)\quad f_m = \frac{(M_X \sin I)^3}{(M_X + M_c)^2} = 6.26 \pm 0.31 M_\odot.$$

The companion star is a K giant with $M \sim M_\odot$, implying that

$$(578)\quad M_x \sin^3 I \sim 6.3 M_\odot$$

and so

$$(579)\quad M_X \gtrsim 6.3 M_\odot.$$

However, from the binary period ($P = 6.4$ d) we find only that

$$(580)\quad a \gtrsim 0.12 \text{ AU}$$

which is far larger than the Schwarzchild radius for a black hole of this mass. Thus it was some time before evidence for V404 Cyg's black hole nature was widely accepted.

Observational evidence for supermassive black holes came initially from the velocity dispersion of stars near the centers of nearby galaxies. More recently, unambiguous evidence for these beasts came from orbital monitoring of stars around Sagittarius A* (in the Milky Way, $M \sim 4 \times 10^6 M_\odot$) and an image of the accretion disk and black hole shadow in the center of M87 ($M \sim 6 \times 10^9 M_\odot$); both are shown in Fig. 50.

22.4 *Non-Newtonian Orbits*

In general, sufficient evidence for a black hole requires demonstrating that too much mass is in too small of a volume, such that the mass much be enclosed within one Schwarzchild radius:

$$(581)\quad R_S = \frac{2GM}{c^2}.$$

But another key sign can be orbits with strongly non-Keplerian features that encode the nature of strong (relativistic) gravity.

Recall that the Keplerian two-body problem (Sec. 2) can be reduced to a

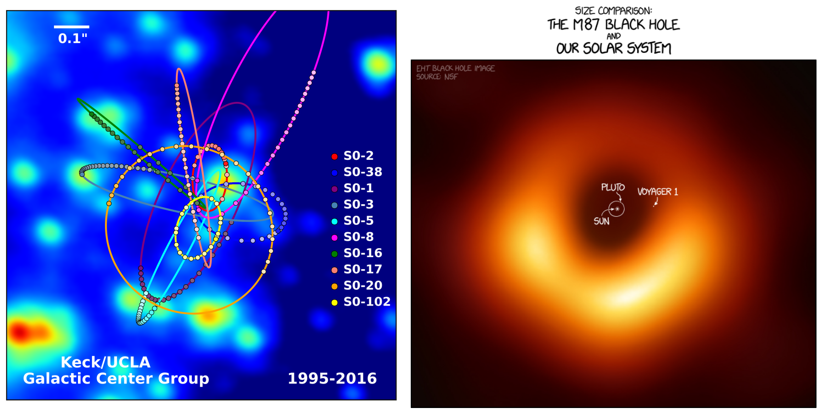

Figure 50: *Left:* Stellar orbits around Sgr A*, the supermassive black hole at the center of the Milky Way. Star S0-2 has a period of 16 yr, while other orbits are longer-period. (From `http://www.astro.ucla.edu/~ghezgroup/gc/`). *Right:* Accretion disk and shadow of the supermassive black hole at the center of nearby galaxy M87. The bright ring's diameter is 42μas, or $\sim 2000\times$ smaller than the scale bar at *left*.

one-dimensional effective potential:

$$E = \frac{1}{2} m \left(\frac{dr}{dt} \right)^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r} \tag{582}$$

or

$$\epsilon = \frac{1}{2}\dot{r}^2 + \frac{l^2}{2r^2} - \frac{GM}{r} \tag{583}$$

$$= \frac{1}{2}\dot{r}^2 + V_{\rm eff} \tag{584}$$

where ϵ and ℓ are the energy and angular momentum per mass, respectively. Fig. 51 recalls this scenario, with different values of ϵ corresponding to unbound, elliptical, or circular orbits.

The equivalent for orbits in general relativity looks more interesting. If we have a non-spinning black hole, then

$$\left(\frac{dr}{dt} \right)^2 = \frac{\epsilon^2}{c^2} \left(1 - \frac{2GM}{rc^2} \right) \left(c^2 + \frac{\ell^2}{r^2} \right) \tag{585}$$

where ϵ and ℓ have the same meanings (but ϵ now includes the full relativistic energy, including rest mass energy). But one can again define a relativistic effective potential,

$$(586)\quad V_{\text{eff,rel}} = \left(1 - \frac{2GM}{rc^2}\right)\left(c^2 + \frac{\ell^2}{r^2}\right).$$

For a particular value of ϵ^2, the orbital dynamics are determined by $V_{\text{eff,rel}}$ (analogously to the Newtonian case). Fig. 51 compares this case to the classical Keplerian case. A few interesting features that distinguish this new scenario:

- Circular orbits still exist if ϵ^2 is tangent to and just touches V_{eff} at a local minimum.
- Now there is an extra "hump" in the profile whose height depends on ℓ. This means that for certain values of ℓ^2, no local minimum exists – and thus in these cases there are no stable circular orbits.
- If ϵ is high enough for a given ℓ, the trajectory can reach $r = 0$ (this never happens in the classical case for nonzero angular momentum). This is a singularity: here tidal forces become infinitely strong, and anything approaching it will be shredded.

The local minimum disappears for

$$(587)\quad \ell = \sqrt{12}\frac{GM}{c}$$

which corresponds to a stable circular orbit at $r = 3R_s$. We therefore expect no orbits inside of this radius. So even inside an accretion disk, we should have

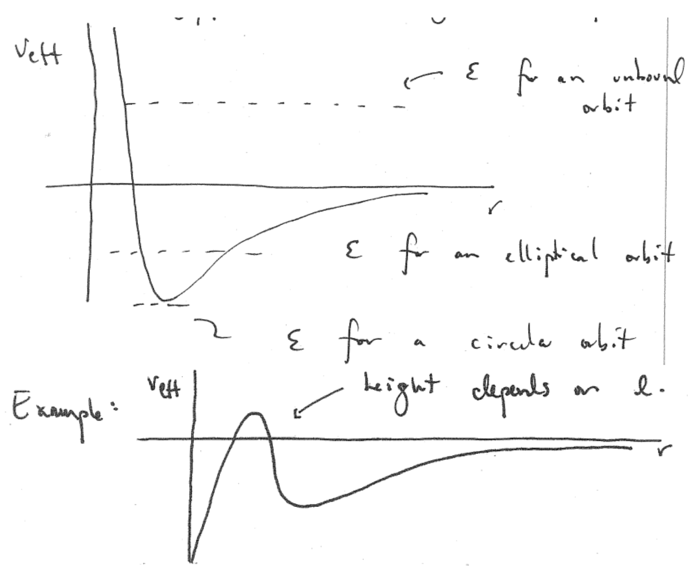

Figure 51: Effective potential vs. separation. *Top:* in a classical, Keplerian two-body system; *Bottom:* in the relativistic limit.

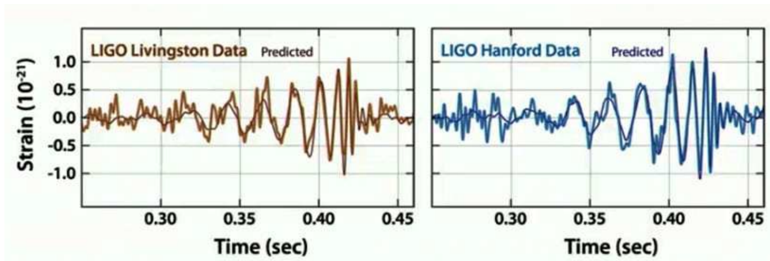

Figure 52: Gravitational wave event GW150914, indicating the inspiral and merger of two black holes.

a hole a few times larger than any black hole's event horizon.

Note that things get even more exciting once we bring rotation into the picture. The spin of a black hole has several interesting effects:

- The event horizon changes size and shape
- Orbits have a much more complicated (non-spherical) potential.
- Orbital frequencies become affected by "frame-dragging" as the spinning black hole twists spacetime around itself.

Thanks to the **no-hair theorem**, it turns out that everything about a black hole (including the orbits around it) can be described by just three parameters: mass, angular momentum (spin), and electric charge.

22.5 *Gravitational Waves and Black Holes*

Black holes must solve the Einstein equations in vacuum, $G_{\mu\nu} = 0$. This is true even if two black holes are close together. In this case, they emit gravitational waves – potentially with a much higher GW luminosity than the neutron star binaries whose inspiral also indicates GW emission (Sec. 21.6). It wasn't until the mid-2000s that computational relativity calculations first predicted what happens when two black holes orbit each other. The result, later spectacularly verified by gravitational wave measurements (see Fig. 52) includes three epochs:

1. **Inspiral:** Long before the merger, the binary is on a nearly-periodic orbit - but energy is being lost due to GW emission, so the semimajor axis (and period) steadily shrinks. Motion here is determined by the effective potential $V_{\text{eff,rel}}$, but with ϵ and ℓ slowly evolving.
2. **Plunge and Merger:** As the gravitational field grows in strength, eventually the orbits become unstable and the binary members rapidly come together, forming a single object.
3. **Ringdown:** A few, last oscillations are seen as the merged remnant settles down to the exact Kerr solution for a rotating black hole (enforcing the no-hair theorem).

This structure matches most of the gravitational wave events found so far (see e.g. Fig. 44). *Only* a black hole model, including all the necessary (very!) strong gravity physics, is able to explain these observations.