2 Size and Distance Scales

Space is big. Really big. So let's consider size scales in astrophysics.

2.1 *Size Scales*

- Bohr radius: the size of a hydrogen atom in Bohr's semiclassical model is $\hbar^2/m_e k_e e^2 \approx 0.5\mathring{A} = 5 \times 10^{-11}$ m.

- Earth radius: Since the French Revolution defined $\pi R_\oplus = 20,000$ km, we have $R_\oplus \approx 6,300$ km$\approx 6.3 \times 10^6$ m.

- Solar radius: a rough rule of thumb is that $R_\odot \sim 100 R_\oplus$, and so $R_\odot \approx 7 \times 10^8$ m. (If you like gas giant planets such as Jupiter, $R_{\rm Jup} \sim 10 R_\oplus \sim 1/10 R_\odot$.)

- Astronomical unit: the distance from the Earth to the Sun. A rough rule of thumb is that 1 au$\approx 200 R_\odot$, and so 1 au$\approx 1.5 \times 10^{11}$ m. (This is also roughly 8 light-minutes.)

- Parsec: the fundamental unit of distance beyond the Solar System (no 'light years' here). Space is really empty: 1 pc$\approx 2 \times 10^5$ au! So 1 pc$\approx 3 \times 10^{16}$ cm.

As shown in Fig. 1 the parsec is observationally defined via **trigonometric parallax**, which is how astronomers measure the distance to the nearest stars. The Earth's orbit has a diameter of 2 au, so over half a year the Earth moves by that amount and an object at distance d will appear to shift position slightly by an angle 2θ. Then we have

$$\tan\theta = \frac{1 \text{ au}}{d}. \quad (10)$$

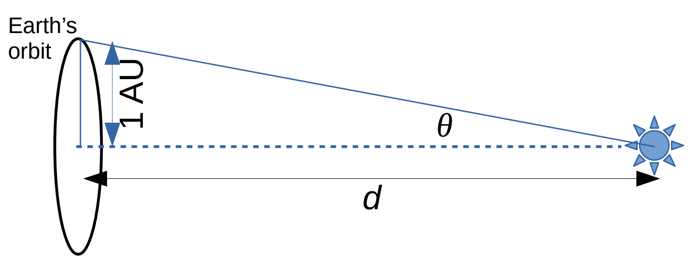

Figure 1: Trigonometric (parallax) distance measurements use the Earth's orbit to measure the changing angular position of distant objects. If the angular movement θ (**parallax**) is measured in arcsec, then the distance d will be measured in parsecs (pc).

Since θ is very small (try calculating a few examples!) when we meaure it in radians we can approximate $\tan\theta \approx \theta$, so we get

$$d = 1 \text{ AU}/\theta \tag{11}$$

or more conveniently

$$\frac{d}{\text{pc}} = \frac{1 \text{ arcsec}}{\theta}. \tag{12}$$

The name **parsec** comes from the distance to an object with a **PAR**allax of one arc**SEC**. It is also (by coincidence) roughly the average distance between stars in the Solar neighborhood. For example, α Centauri (the nearest star system) is 1.3 pc away. To the center of our Milky Way galaxy, though, is roughly 8,000 pc (8 kpc), and to the next-nearest galaxy of reasonable size is 620 kpc! Space is big.

2.2 *Cosmic Distance Ladder*

Distance is a key concept in astrophysics, and until recently often one of the least-precisely-known quantities we can measure. (Since ~2015 this has begun to change somewhat thanks to ESA's **Gaia** mission, which is measuring trigonometric parallax for billions of objects with sub-milliarcsec precision – i.e., precision of $< 10^{-3}$ arcsec.

The **distance ladder** refers to the bootstrapping of distance measurements, from nearby stars to the furthest edges of the observable universe. It's a 'ladder' because each technique only has a limited range of applicability, as described below. Note that inside the Solar System we can just throw EM waves at something and wait for them to bounce back: the round-trip light travel time gives us the distance. This includes planetary radar, spacecraft communication, laser-ranging to the moon, and other examples; this isn't usually considered part of the distance ladder.

Parallax, already mentioned, is the first rung outside the Solar system. As noted this is being measured by Gaia for ~1 billion stars across ~half of the Galaxy. A revolution is underway!

Standard candles are objects with known intrinsic brightness (i.e., luminosity). If we know how bright something is, then we also know its absolute magnitude. By Eq. 3, it's easy to calculate its distance. There are relatively few truly standard candles, but a larger array of objects are standardizable: using other information we can correct for intrinsic variations between different objects. Some examples include:

- **Cepheid variables**: giant pulsating stars, whose pulsation period correlateds with luminosity.
- **Type Ia Supernovae**: one form of exploding stars (probably white dwarf)
- Distant Galaxies: it turns out that when we measure the rotation speed v_{rot} of various galaxies, we often find $L_{\text{gal}} \propto v_{\text{rot}}^2$: so measuring v_{rot} gives us the galaxy's intrinsic brightness.