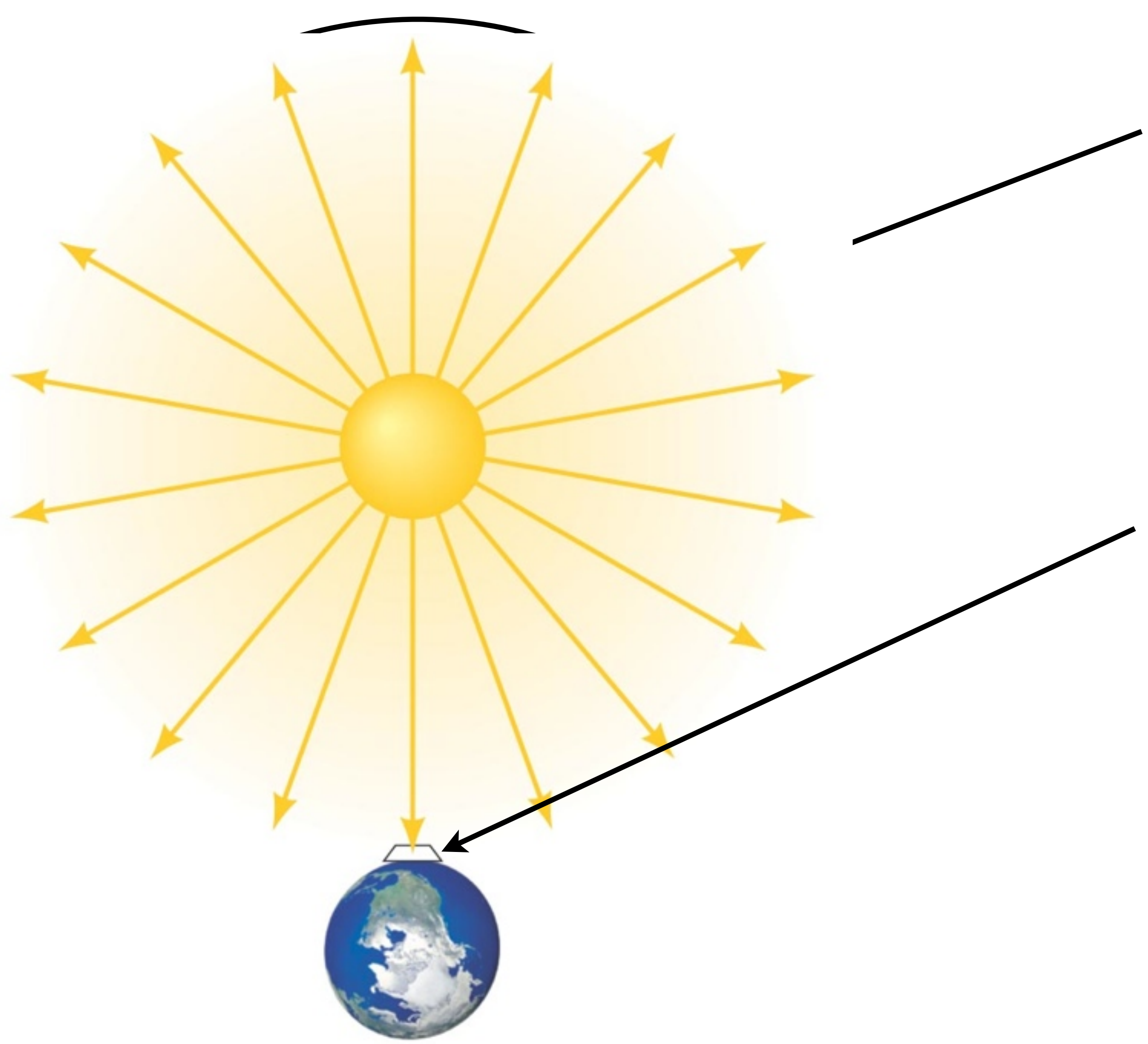
Intensity vs. luminosity

- flux(f) - how bright an object appears to us. Units of **[energy/t/area]**. The amount of energy hitting a unit area.
- luminosity (L) - the total amount of energy leaving an object. Units of **[energy/time]**



Total energy output of a star is the luminosity

What we receive at the earth is the apparent brightness.

Not to scale!

What we will cover today

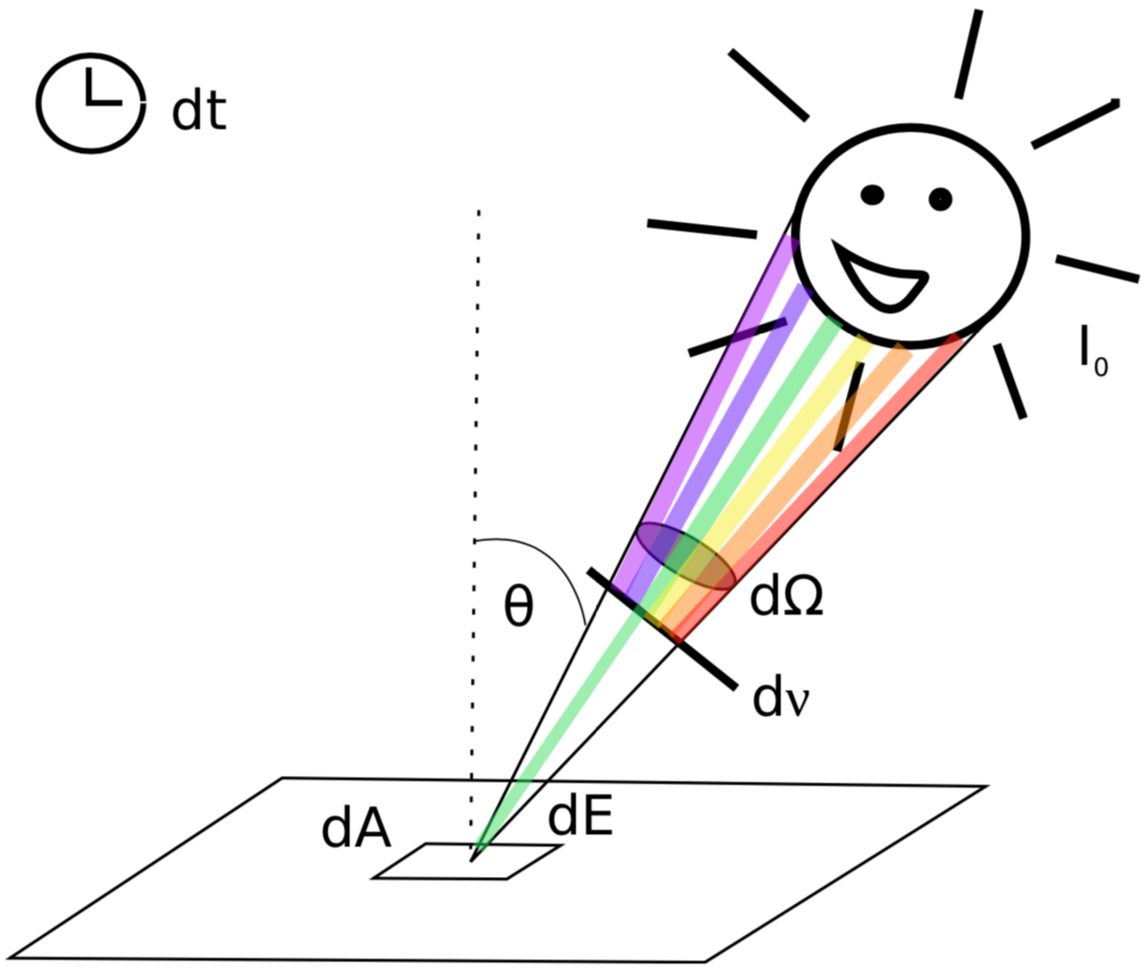
- The brightness of objects
  - Intensity
  - Flux
  - Luminosity
  - How they all relate
- The relation between flux, luminosity and distance
- The total emission of a blackbody
- The spectrum of a blackbody

Different ways to measure light coming from an object

- What are the different parameters that we have to consider in the diagram below?
- Need to consider the amount of light that leaves the object with a frequency between **ν and ν+dν** as **$I_{0,\nu}$**
- From an observer with area **dA** we see light coming from direction **θ** away from the normal to dA.
- The source covers a solid angle **dΩ** and has The light is measured in a given time interval **dt**
- The total energy received is

$$dE_\nu = I_{0,\nu} \cos\theta \; dA \; d\Omega \; d\nu \; dt$$

specific intensity



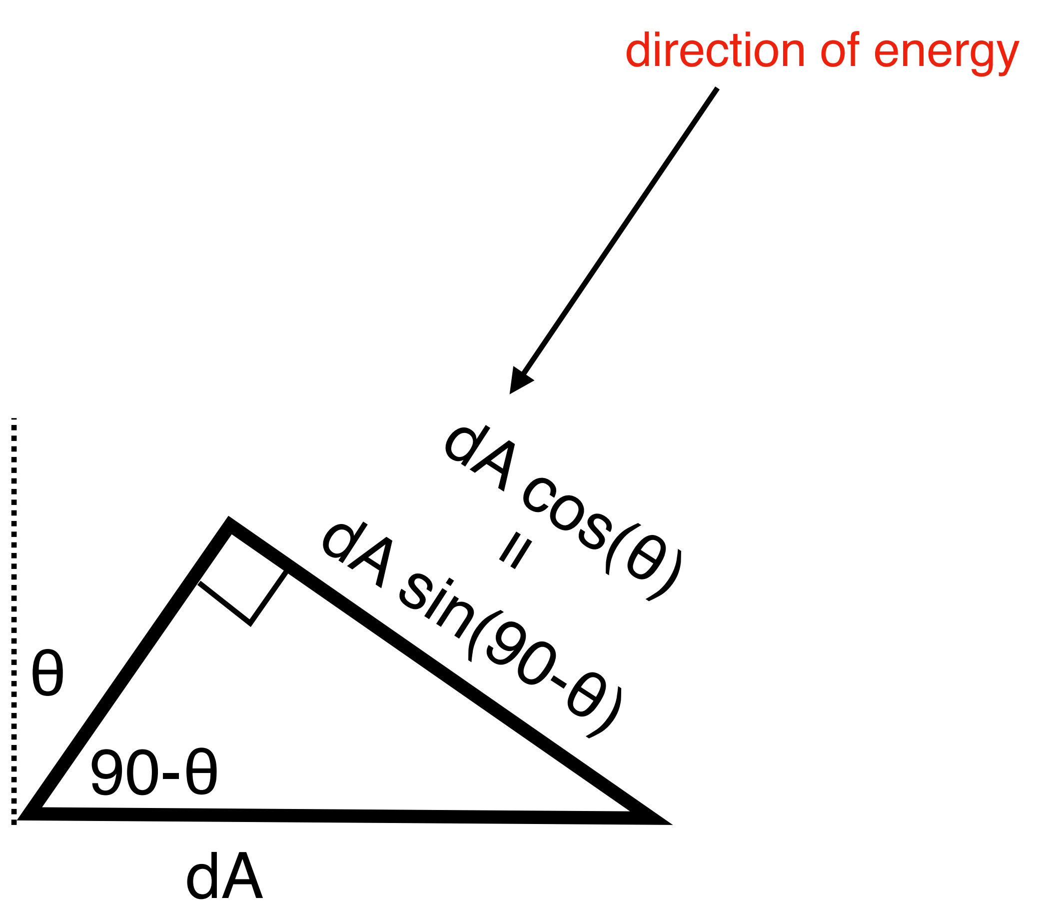
Intensity

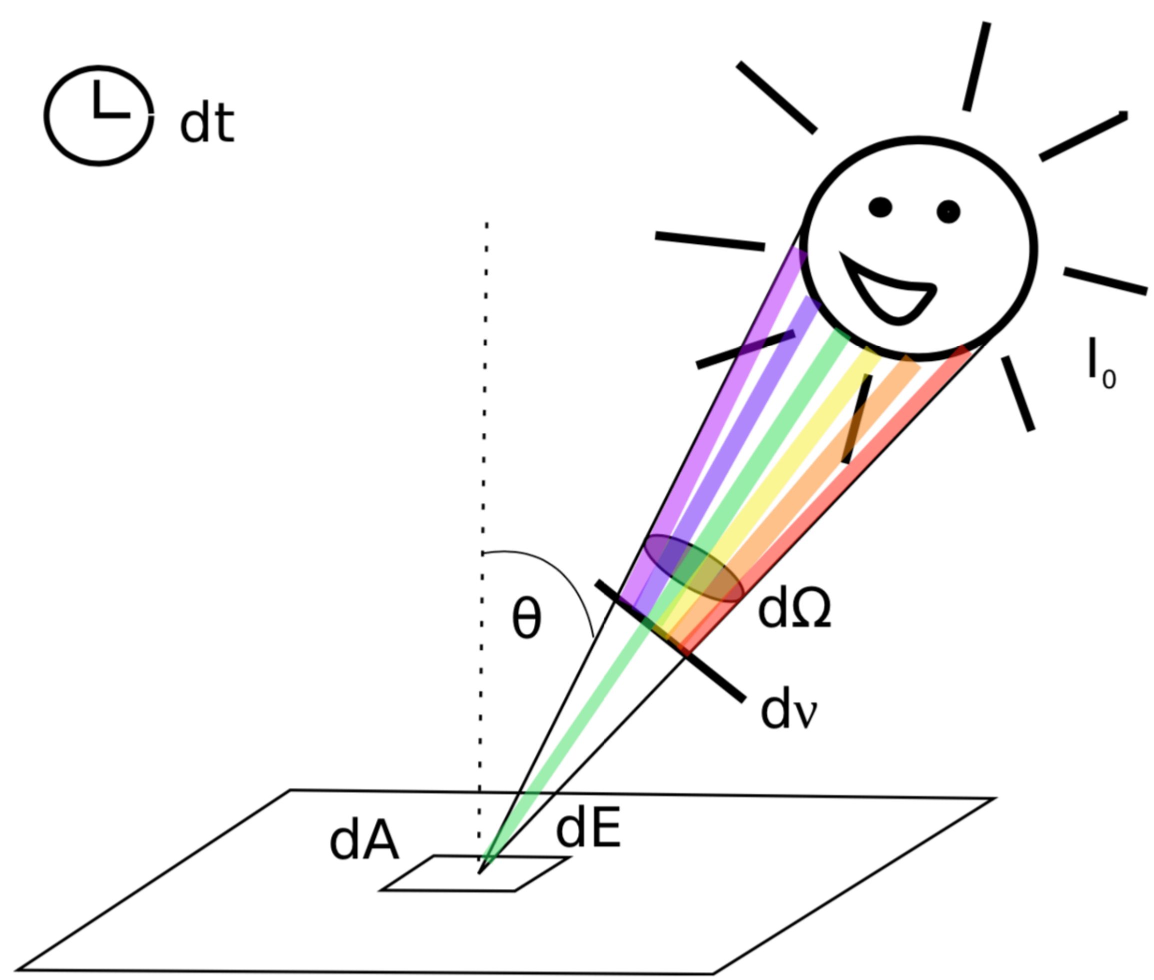
- The total energy received from an angular area of the object

  intensity = $dE_\nu = I_{0,\nu} \cos\theta \, dA \, d\Omega \, d\nu \, dt$ The units of this are [J s^{-1} Hz^{-1} m^{-2} sr^{-1}]

- What is "cos θ dA" term for?

- Energy received is perpendicular to incident direction





Flux density and bolometric flux

- The total energy received from an angular area of the object

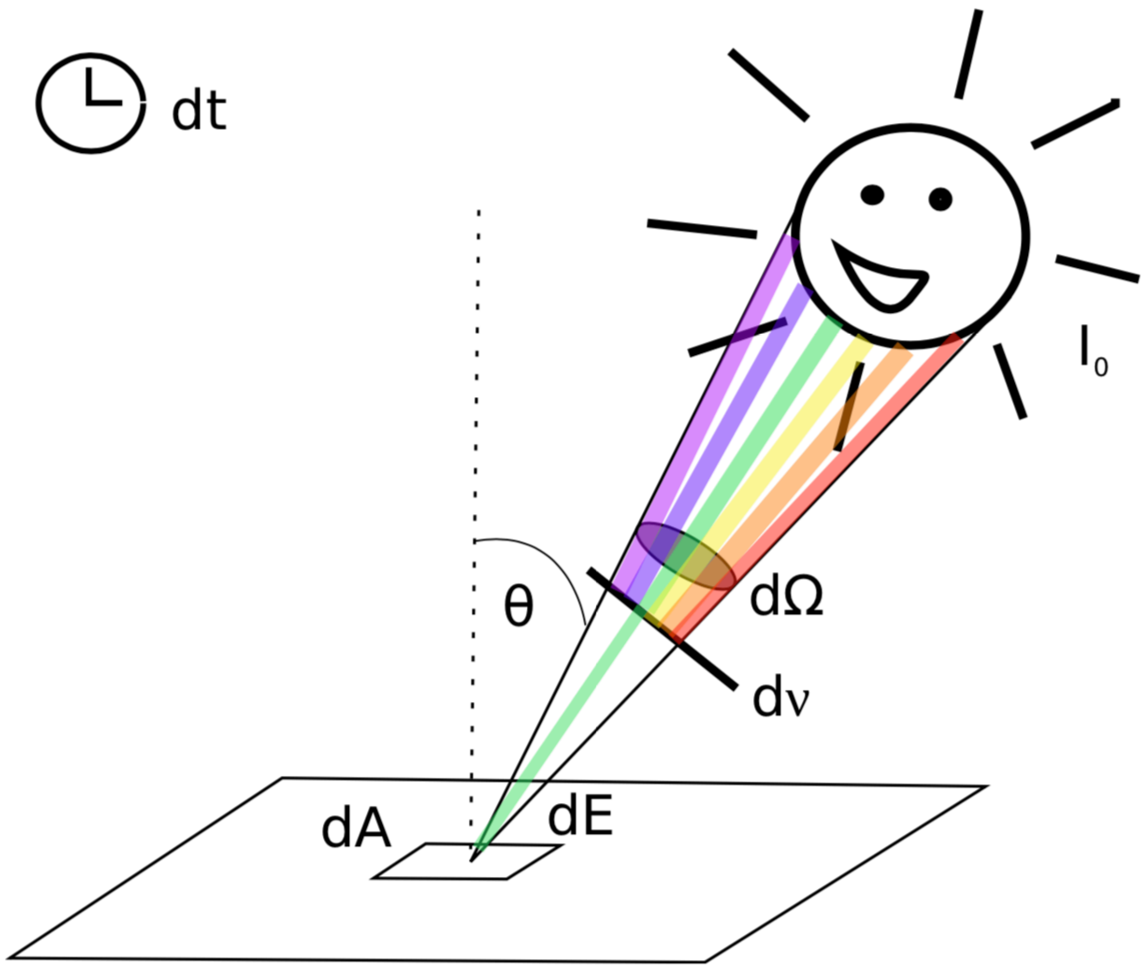
  intensity = $dE_\nu = I_{0,\nu} \cos\theta \; dA \; d\Omega \; d\nu \; dt$ The units of this are [J s^{-1} Hz^{-1} m^{-2} sr^{-1}]

- **Flux density** is the total energy integrated over the solid angle of a source, per unit area, per unit time, per unit frequency

$$F_\nu = \int_\Omega \frac{dE_\nu}{dA \; dt \; d\nu} = \int_\Omega I_\nu \cos\theta \; d\Omega$$

- What are the units of F_ν?
- [J s^{-1} Hz^{-1} m^{-2}]
- **Bolometric flux** is the flux over all frequencies

$$F = \int_\nu F_\nu d\nu$$ with units [J s^{-1} m^{-2}]



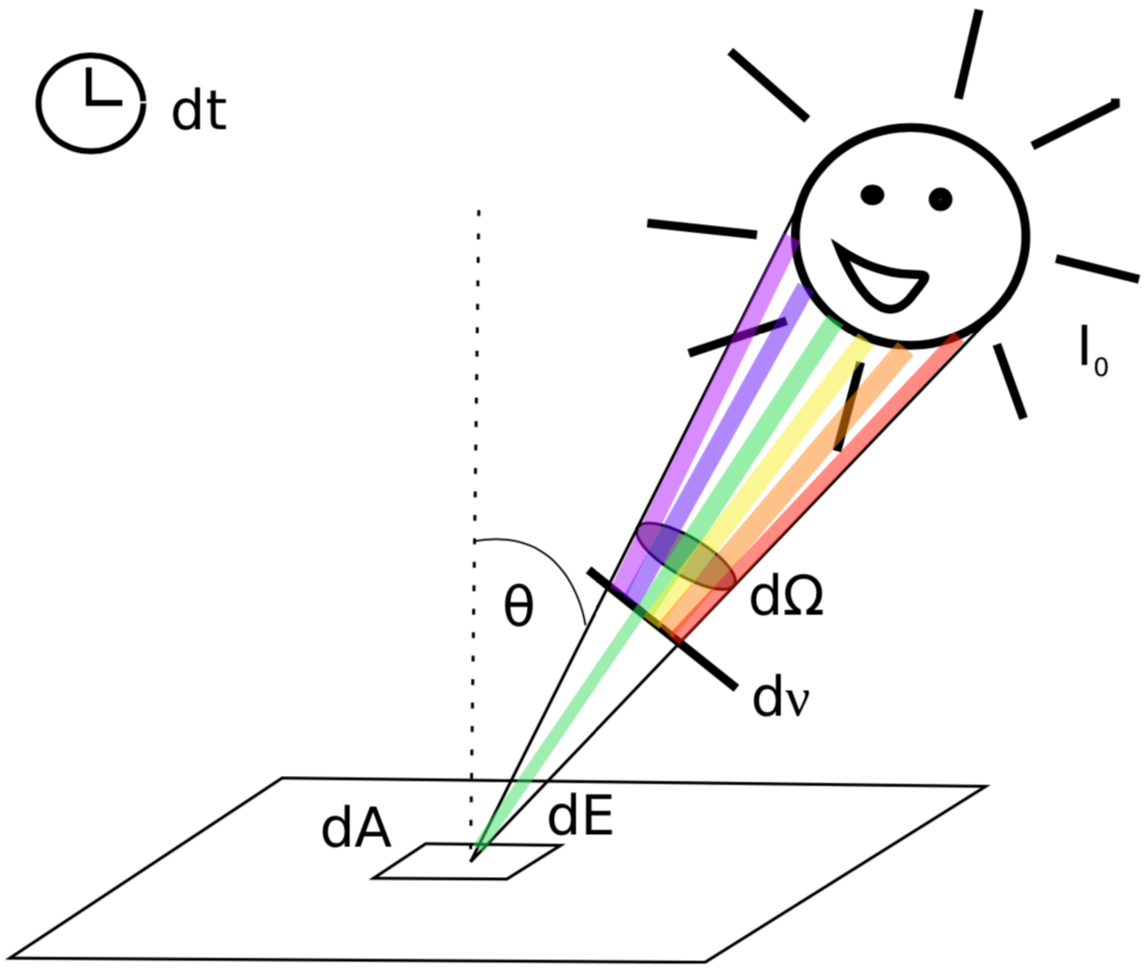
Flux vs luminosity

- **Bolometric flux** is the flux over all frequencies

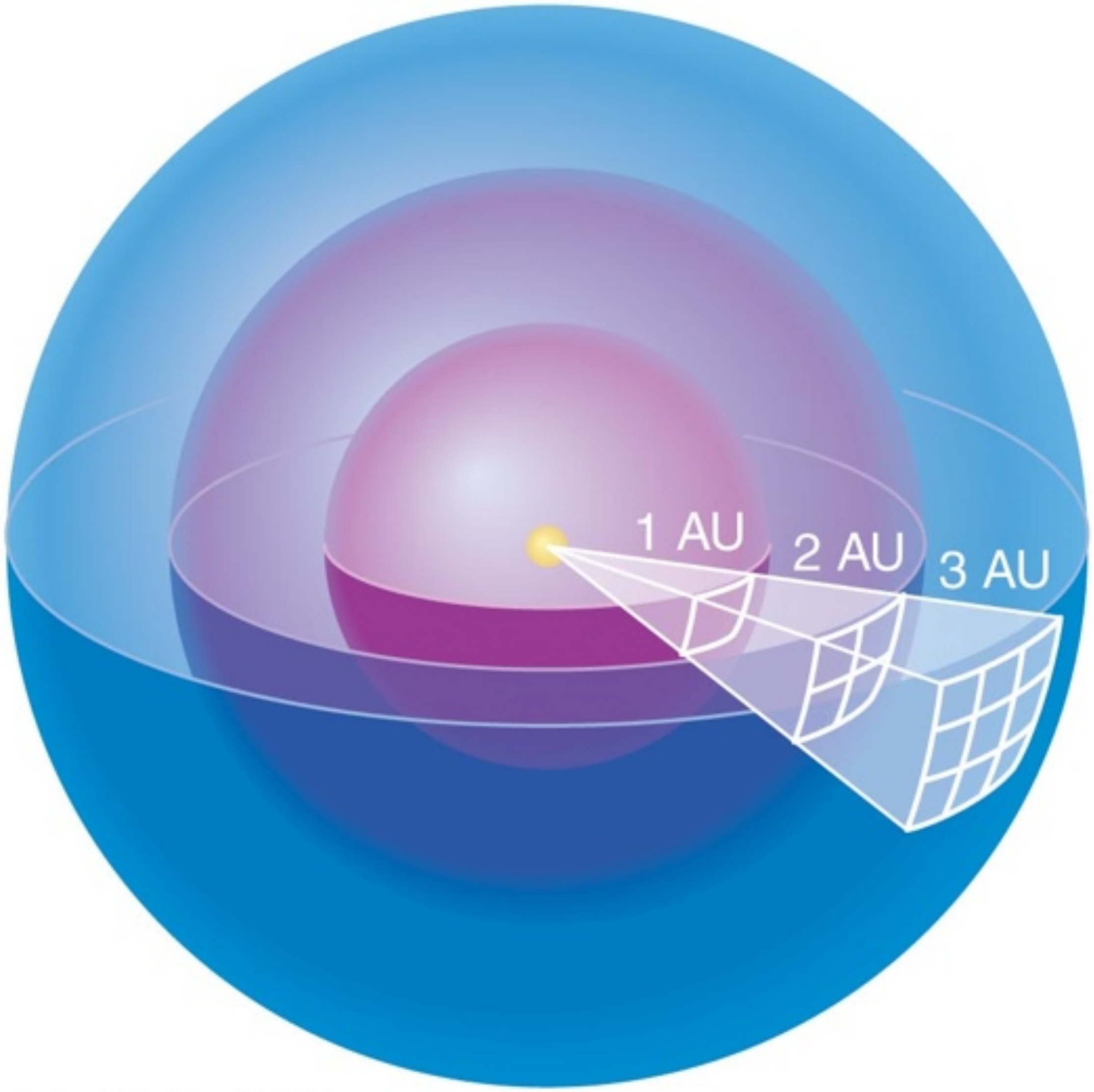
$$F = \int_\nu F_\nu d\nu$$ with units [J s^{-1} m^{-2}]

- **Luminosity** is the flux integrated over all areas

$$L = \int F\, dA$$ and has units of [J s^{-1}]



The dependence of apparent brightness on distance: The inverse square law



$$L = \int F \, dA$$

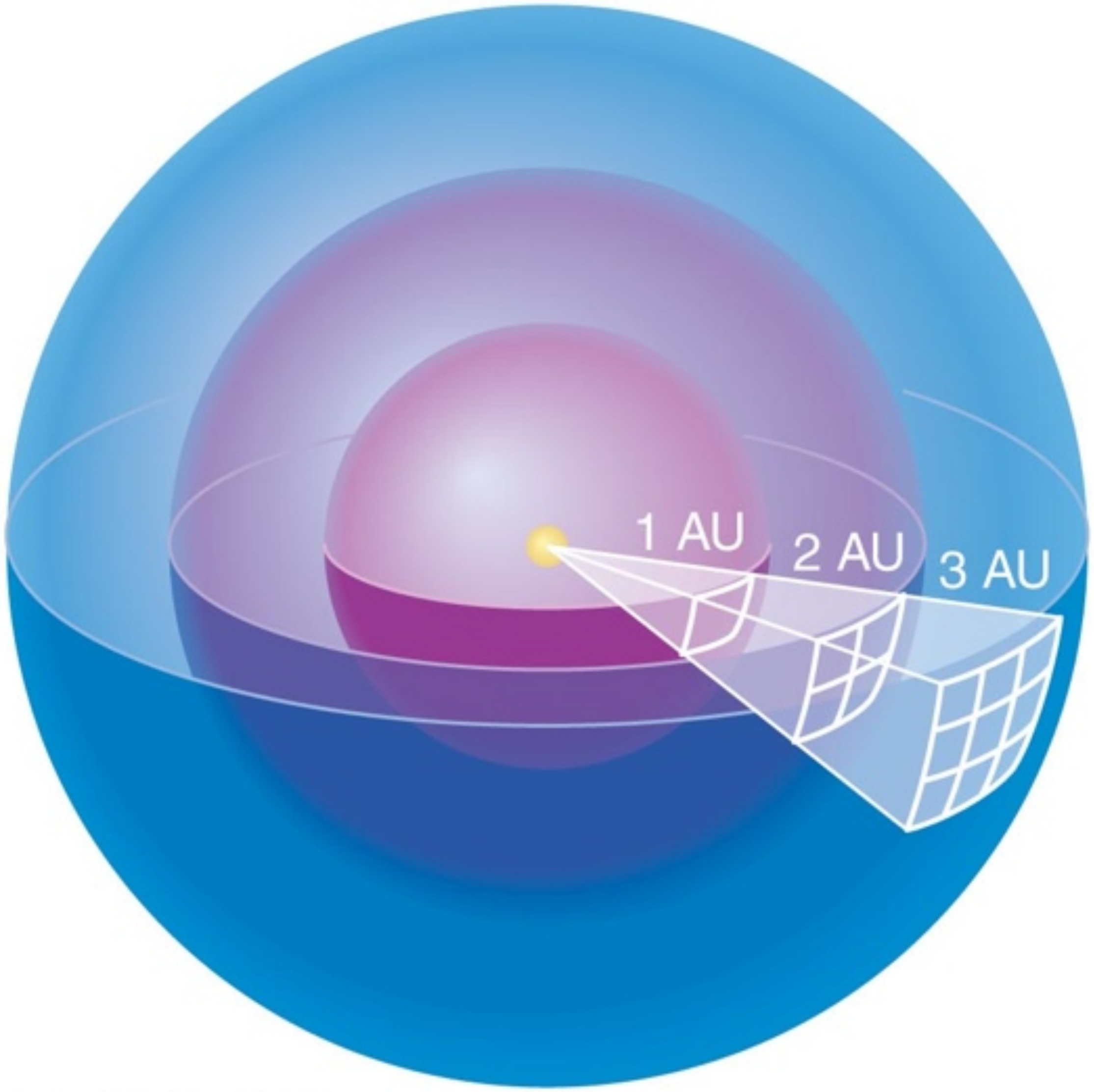
If we consider that luminosity collects all the light over a sphere, we can relate the flux to luminosity using geometry

The **total** amount of light coming out of an object does not change with distance.

The amount hitting a fixed area *(like your camera lens or eye)* decreases with distance.

$$F = \frac{L}{4\pi d^2} \Rightarrow L = 4\pi d^2 F$$

The dependence of apparent brightness on distance: The inverse square law



$$F = \frac{L}{4\pi d^2} \Rightarrow L = 4\pi d^2 F$$

If a source has a luminosity of $1L_{\odot}$=3.826x10^{26} W and is at a distance of 3 Ly, what is the flux?

$$3 \text{ Ly} = 2.83 \times 10^{16} \text{ m}$$

$$F = \frac{3.826 \times 10^{26} \text{ W}}{4\pi(2.83 \times 10^{16} \text{ m})^2} = 3.78 \times 10^{-8} \text{W m}^{-2}$$

The sun at 3 Ly is very faint!

1 Ly = 9.461x10^{15}m

The simplest kind of emitting object

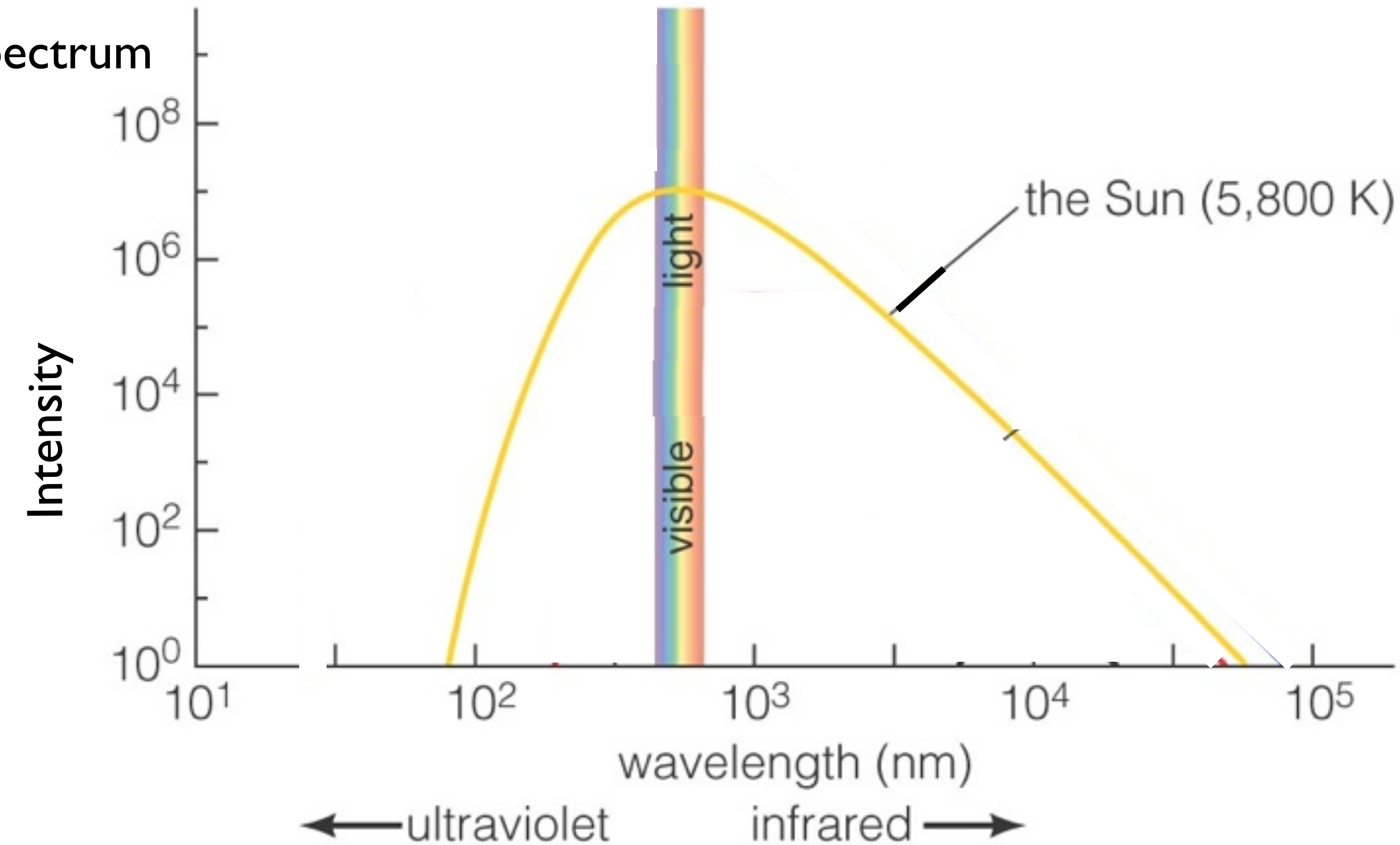
"**spectrum**" is a graph of an objects intensity as a function of wavelength.

"**Blackbody**" is an object that is dense, absorbs all light that hits it, and remits that light with a spectrum that depends on the objects temperature.

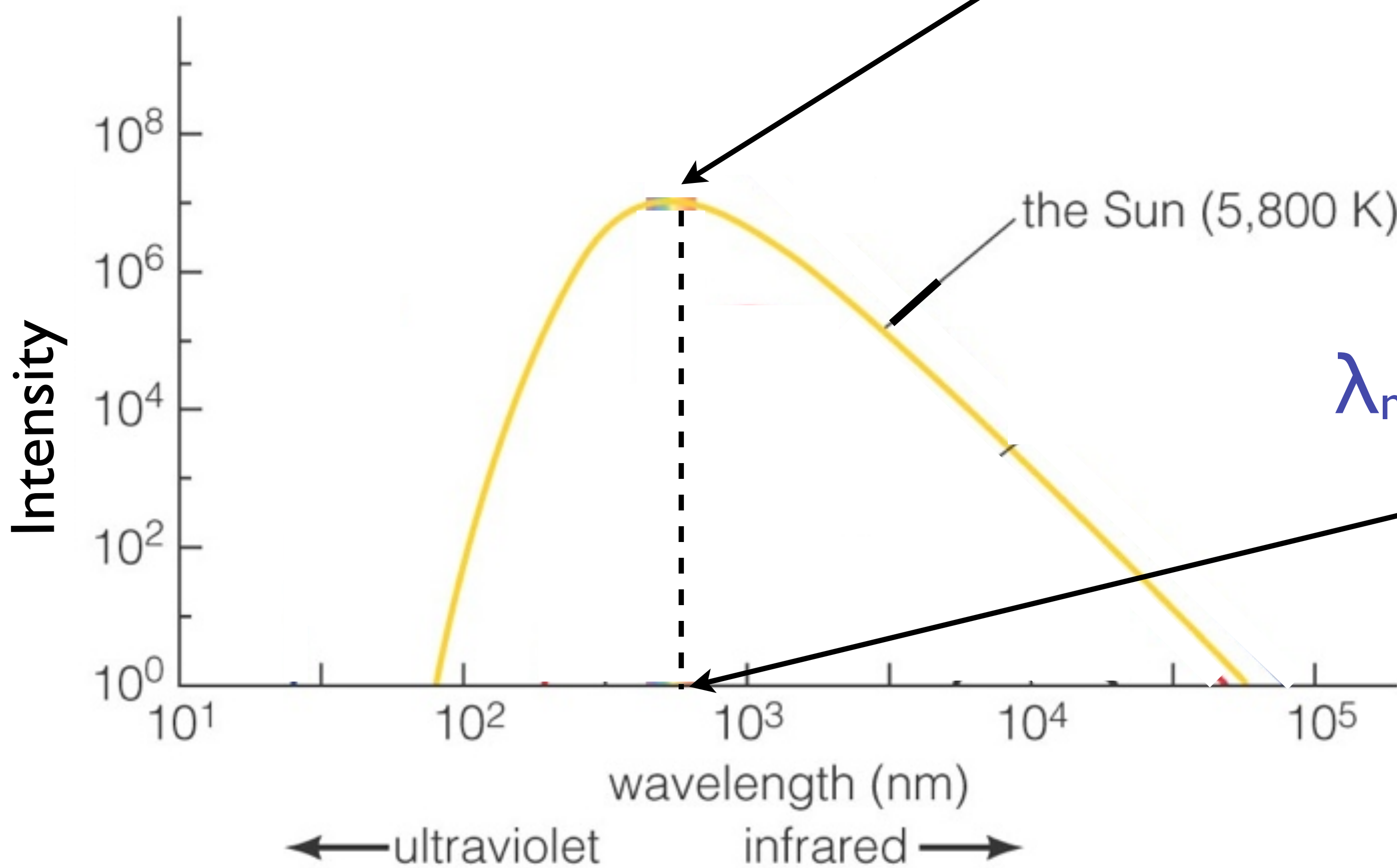
"**Blackbody curve**" is the spectrum of a blackbody.

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

B_λ has units W m^{-2} Hz^{-1} sr^{-1} or energy emitted per time per unit surface area per solid angle.



peak of blackbody curve



Wien's law

$$\lambda_{max}[m] = (2.89 \times 10^{-3}\ K\ m)\ /\ T[K]$$

The **WAVELENGTH** that the **PEAK** of the blackbody curve occurs at tells us about the object's **TEMPERATURE** and **COLOR**.

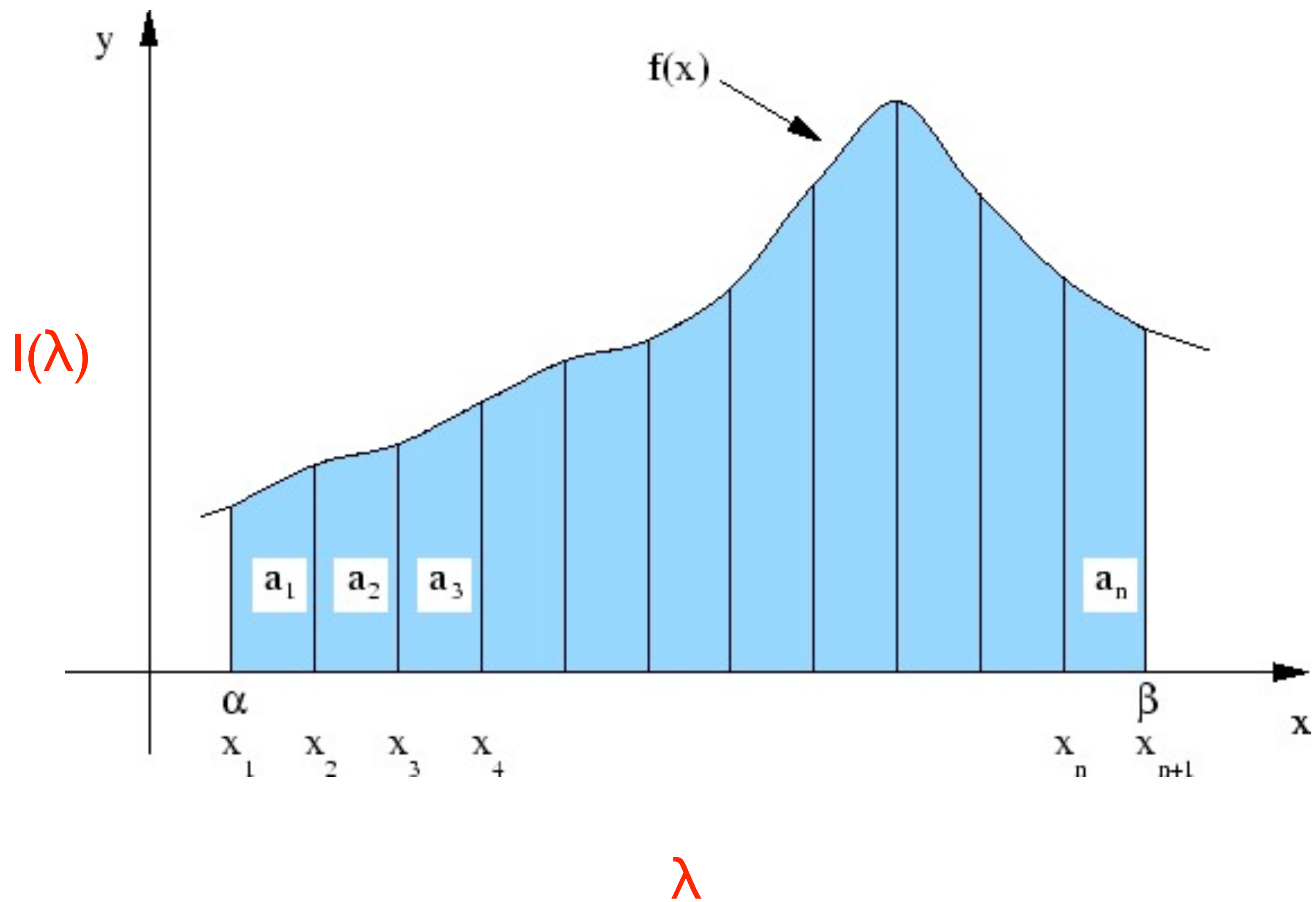
The curve behaves differently on both sides of this peak

Integrating a curve

The total intensity is given by

Integrating a curve gives you the area under the curve.

The integral of a spectrum gives you the total intensity of an object over the wavelength range of the integral.



$$f_{tot} = \int_{\lambda_1}^{\lambda_2} f_\lambda(\lambda) d\lambda$$

Stefan-Boltzmann Law

- Describes the total amount of energy emitted by a patch of surface on a blackbody.
- $F = (5.67\times10^{-8}\ J/s/m^2/K^4)\ T^4$
- If object A has T_A=100K and object B has T_B=200K, how much more energy per m^2 does object B emit?
- Object B emits 16 times more energy than object A
- Luminosity is how much total energy an object emits.
- The Luminosity (L) depends on an object's surface area (A) and temperature (T).
- $L=(5.67\times10^{-8}\ J/s/m^2/K^4)(A)(T^4)$
- For a spherical object with radius R
- $L=(5.67\times10^{-8}\ J/s/m^2/K^4)(4\pi R^2)(T^4)$
- **So, BIGGER and hotter objects are brighter than** smaller **and cooler objects.**

The Stefan-Boltzmann law

- what is the relative luminosity of the objects A and B if:

$T_A = 100\text{K}$ $T_B = 200\text{K}$

$R_A = 10\text{m}$ $R_B = 5\text{m}$

$$\frac{L_A}{L_B} = \frac{T_A^4 R_A^2}{T_B^4 R_B^2} = \frac{(100\ \text{K})^4 (10\ \text{m})^2}{(200\ \text{K})^4 (5\ \text{m})^2} = \frac{10^{10}\text{K}^4\ \text{m}^2}{4 \times 10^{10}\text{K}^4\ \text{m}^2}$$

The Stefan-Boltzmann law

- $L_A = L_B = 10^4$ J/s; $R_A = 10^4$m; $R_B = 10^5$m. Which star has the greater temperature?

$L_A = 10^4$ J/s
$R_A = 10^4$m

$L_B = 10^4$J/s
$R_B = 10^5$m

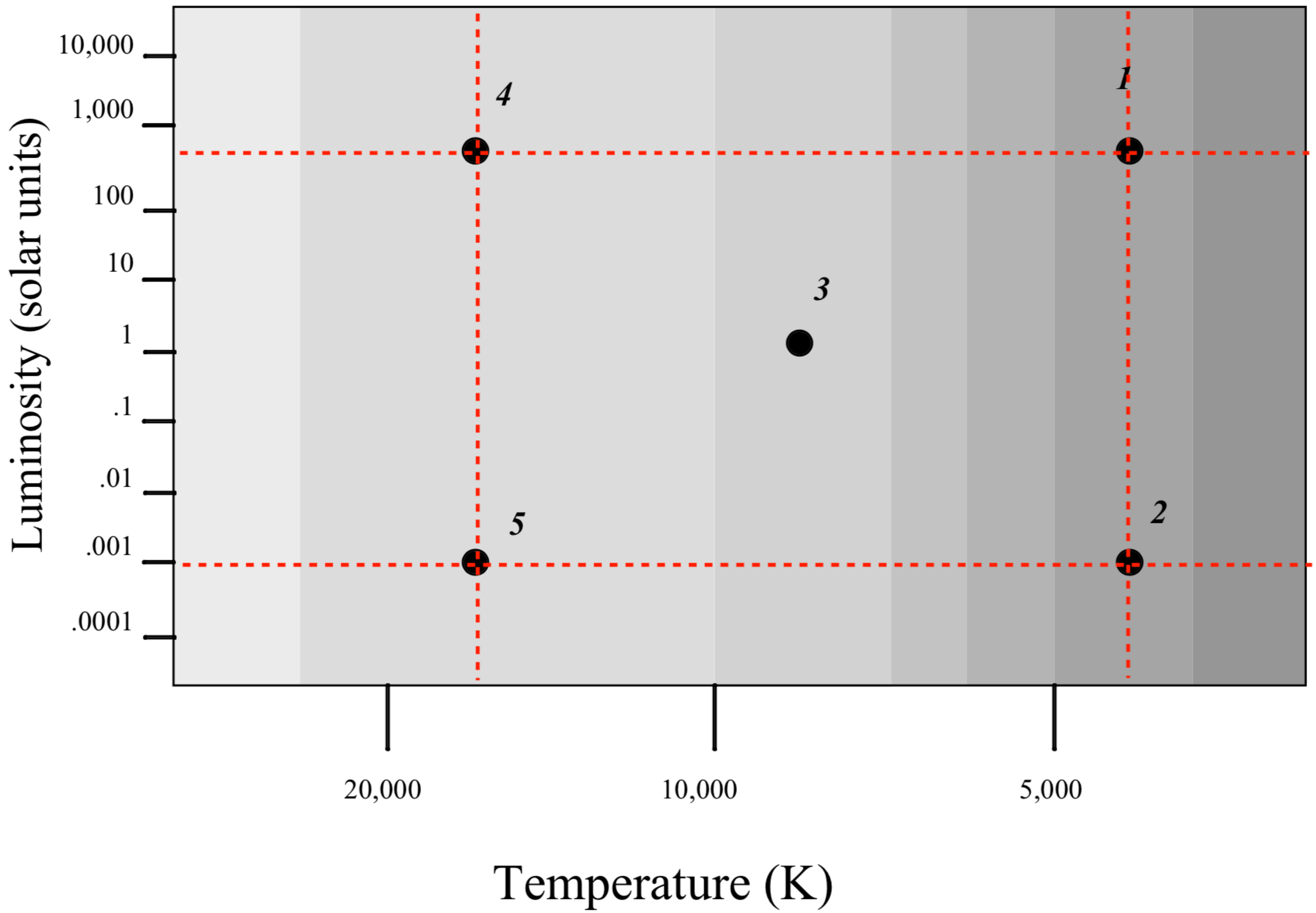
$$T = \left(\frac{L}{R^2}\right)^{1/4} \Rightarrow \frac{T_A}{T_B} = \left(\frac{L_A}{R_A^2}\frac{R_B^2}{L_B}\right)^{1/4}$$

$$\frac{T_A}{T_B} = \left(\frac{10^4 \text{ J s}^{-1}}{(10^4 \text{ m})^2}\frac{(10^5 \text{ m})^2}{10^4 \text{ J s}^{-1}}\right)^{1/4} = \left(\frac{10^{10} \text{ m}^2}{10^8 \text{ m}^2}\right)^{1/4} = 100^{1/4}$$

A must be hotter because it has the same luminosity at a smaller radius

Which star is the largest?

Which star is the smallest?



$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

$$log\ B_\lambda(T) = log\left(\frac{2hc^2}{\lambda^5}\right) + log\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)^{-1}$$

$$= log\ 2hc^2 \underbrace{- 5\ log\lambda}_{\text{blue}} \underbrace{- log\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)}_{\text{red}}$$

This function behaves differently in different regimes.
For a fixed T:

- if $\lambda >> \lambda_{max}$ **then middle term** gets rapidly smaller while **last term** only slowly grows. So **log B_λ α -5 log λ**, or a straight line in log space
- If $\lambda << \lambda_{max}$ then **last term** dives to exponentially fast while the **first term** only slowly rises. So **log B_λ** has an exponential cutoff.

When you see a function, think about its behavior!