19 Nuclear Reaction Pathways

19.1 *Useful references*

- Choudhuri, Sec. 4.3
- Kippenhahn, Weiger, and Weiss, 2nd ed., Secs. 18.5–18.6
- Hansen, Kawaler, and Trimble, Secs. 6.3–6.8

19.2 *First fusion: the p-p chain*

For stars with $M \lesssim 1 M_\odot$, the dominant fusion pathway builds ${}^4_2$He nuclei (also known as "α" or **alpha particles**) from individual protons. This typical Helium nucleus has two protons and two neutrons, yet it is built up from protons alone through a series of nuclear reactions termed the **p-p chain** (for "proton-proton" chain). The overall reaction can be described as

(409) $4p \rightarrow^4_2 \mathrm{He} + 26.4 \ \mathrm{MeV}.$

The energy released is roughly 0.7% of the mass energy of the individual protons (that is, 26 MeV / ($4 m_p c^2$) ≈ 0.007).

Why is energy released at all in a nuclear reaction? Because the ingredients are more massive (and so have more mass energy) than the products. In the case of the reaction above, we can look up that a proton has $m_p = 1.0073$ atomic mass units (amu) or $m_p c^2 = 938.3$ MeV, while the helium nucleus produced has a mass of 4.0026 amu. So the ingredients in this reaction (4 protons) have mass energy 4×1.0073 amu=4.0292 mu, more than the mass in the produced helium. Where did this mass go? It was converted into energy and released as a combination of radiation, kinetic energy of the particles involved, and as speedy, lightweight neutrinos. The ratio of product mass over ingredient mass is $\approx$0.993, and so we often say that fusing H into He releases about 0.7% of the available mass energy.

We've already encountered the first, weakest, and rate-limiting step in this process, namely

(410) $p + p \rightarrow^2_1 \mathrm{H} + e^+ + \nu.$

As we saw in the preceding chapter, the cross-section for this inverse beta-decay reaction is so low that a given proton will only undergo it in a $\sim$few Gyr. The next step,

(411) $p +^2_1 \mathrm{H} \rightarrow^3_2 \mathrm{He} + \gamma$

happens very quickly – it takes only about 1.4 s in the Sun.

After ^{3}He is produced, there are three different pathways to ^{4}He. These are termed pp1, pp2, and pp3. You might think that in a H-dominated universe (where protons, the nuclei of hydrogen atoms, are plentiful) we could proceed via

(412) $p +^3_2 \text{He} \rightarrow^4_3 \text{Li}$

and another inverse beta decay, but ^{4}Li is highly unstable and its dominant decay mode is the emission of a proton, so that gets us nowhere.

Instead, we have to use larger building blocks and build up via collisions of $^3_2$He. If we have a paucity of $^4_2$He (as was the case shortly after the Big Bang), then we must use the **pp1 pathway**:

(413) $^3_2\text{He} +^3_2 \text{He} \rightarrow^4_2 \text{He} + 2p$

which produces roughly 70% of the total Solar luminosity.

If $^4_2$He is available (and especially at higher temperatures) the **pp2** and **pp3** pathways will dominate. These both begin via

(414) $^3_2\text{He} +^4_2 \text{He} \rightarrow^7_4 \text{Be} + \gamma$

and then branch off. In the Sun, almost all the rest of the luminosity comes from the pp2 pathway,

(415)
$$^7_4\text{Be} + e^- \rightarrow^7_3 \text{Li} + \nu_e$$
(416)
$$^7_3\text{Li} + p \rightarrow^4_2 \text{He} +^4_2 \text{He}.$$

Whereas $< 1\%$ of the Sun's power comes from the pp3 pathway:

(417)
$$^7_4\text{Be} + p \rightarrow^8_5 \text{B} + \gamma$$
(418)
$$^8_5\text{B} \rightarrow^8_4 \text{Be} + e^+ + \nu_e$$
(419)
$$^8_4\text{Be} \rightarrow^4_2 \text{He} +^4_2 \text{He}.$$

As noted above, all three pp chains convert protons into $^4_2$He and so all release the same amount of energy, 26.4 MeV per α particle. But not all of that energy goes into heating the star (and ultimately to observable electromagnetic radiation): an appreciable fraction can be carried away by the neutrinos. The neutrino produced by the pp3 decay of $^8_5$B can carry away up to 15 MeV (with an average of more like 7 MeV).

As a final aside, these pp3 neutrinos are energetic enough that they can be detected via

(420) $^{37}_{17}\text{Cl} + \nu_e \rightarrow^{37}_{18} \text{Ar} + e^-.$

The argon produced is radioactive, and its decay can be easily detected. Neutrinos have tiny interaction cross-sections, but the chlorine needed to capture the neutrino is cheap in bulk – it's historically used in dry-cleaning.

19.3 *The triple-α process*

Via the several p-p chains, stars build up helium nuclei (α particles) from elementary hydrogen nuclei (protons). Many heavier elements now populate the universe that weren't present immediately after the Big Bang. How were they created? The trouble is that both of the next two most likely reactions are endothermic, removing energy from the star instead of contributing to it:

(421) $^4_2\text{He} +^4_2 \text{He} \rightarrow^8_4 \text{Be} -\mathbf{94keV}$

and

(422) $^4_2\text{He} + p \rightarrow^5_3 \text{Li} -\mathbf{2MeV}.$

As with the first step in the pp chain, the solution turns out to be a relatively rare reaction; nonetheless, it is the most effective pathway available. That is the **triple-alpha process**, in which three $^4_2$He nuclei (i.e., alpha particles) interact almost simultaneously, forming a $^{12}_6$C nucleus. This reaction was predicted long before the reaction was known to be feasible, just because there was no other good explanation for the formation of heavier elements; it involves a resonance in the triple-α cross-section that allows this to proceed. The reaction therefore proceeds as

(423) $3\,^4_2\text{He} \rightarrow^{12}_6 \text{C} + 7.5\text{MeV}.$

Note that whereas the pp chain has a mass-to-energy conversion efficiency of 0.7%, the triple-α process is an order of magnitude less efficient ($\sim 0.07\%$). So to support a star of given mass and temperature, the triple-α process would have to burn through mass $\sim 10\times$ faster (this is why red giants and later-stage stars don't live nearly as long as stars on the main sequence). This new reaction pathway also turns out be highly temperature-sensitive, with

(424) $\epsilon_{3\alpha} \propto T^{40}$

when temperatures approach $\sim 10^8$ K.

19.4 *On Beyond ^{12}C*

Once we have ^{12}C, multiple additional pathways open up for the stellar nucleosynthesis.

The CNO cycle

The first of these, the **CNO cycle**, is an alternative to the pp chains for producing $^4_2$He from protons. However, the process here is rather less straightforward and requires the $^{12}_6$C as a kind of catalyst. Multiple variants exist, but all rely on C to produce intermediate isotopes of N and O that are then broken back down to C in the production of an α particle. One common CNO cycle proceeds as follows:

(425)
$$^{12}_{6}\mathrm{C} + ^{1}_{1}\mathrm{H} \rightarrow ^{12}_{7}\mathrm{N} + \gamma$$

(426)
$$^{13}_{7}\mathrm{N} \rightarrow ^{13}_{6}\mathrm{C} + e^{+} + \nu_e$$

(427)
$$^{13}_{6}\mathrm{C} + ^{1}_{1}\mathrm{H} \rightarrow ^{14}_{7}\mathrm{N} + \gamma$$

(428)
$$^{14}_{7}\mathrm{N} + ^{1}_{1}\mathrm{H} \rightarrow ^{15}_{8}\mathrm{O} + \gamma$$

(429)
$$^{15}_{8}\mathrm{O} \rightarrow ^{15}_{7}\mathrm{N} + e^{+} + \nu_e$$

(430)
$$^{15}_{7}\mathrm{N} + ^{1}_{1}\mathrm{H} \rightarrow ^{12}_{6}\mathrm{C} + ^{4}_{2}\mathrm{He}$$

The CNO cycle does not become highly effective until temperatures are somewhat higher than in the Sun's core. Thus at present only 1–2% of the Sun's He comes from the CNO cycle. But this ratio will increase as the Sun evolves and its central temperature steadily increases. Interestingly, the conversion of $^{14}_{7}$N is the slowest step and so nitrogen will tend to build up during this process; the CNO cycle is thus the source of most N.

Alpha-process and higher-order nucleosynthesis

With a sufficient abundance of $^{12}_{6}$C and $^{4}_{2}$He, **alpha-process nucleosynthesis** can begin and heavier elements can be rapidly produced. This requires higher temperatures and densities (and also higher $^{12}_{6}$C abundances) than are found in the Sun's core, but the process becomes the dominant central energy source in the most massive stars. These reactions proceed in a much more straightforward manner than does the CNO cycle or p-p chains:

(431) $$^{4m}_{2m}\mathrm{X} + ^{4}_{2}\mathrm{He} \rightarrow ^{4n}_{2n}\mathrm{Y}$$

for $n = m + 1$, n and m both integers ≥ 3.

^{16}O can also be built up in this way (though some species such as ^{20}Ne are unstable, and so won't be). Other, related fusion processes also occur in stars that are more massive than the Sun. These are termed "______ burning," where you can fill in the blank with your favorite choice of ^{12}C, ^{16}O, ^{32}Si. Regardless of the specific pathway, Eq. 378 suggests that we will rapidly run out of road as we approach ^{56}Fe because it has the highest binding energy per nucleon.

As noted above for the triple-alpha process, each step in the nuclear burning chain becomes progressively less energy efficient. Fusing H→He converts 0.7% of mass into energy, He→C converts just 0.1%, and fusing C→Fe — the end of the line — converts only another 0.1%.