UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2020

Problem Set 3
Due: Wednesday, February 19, 2019, in class
This problem set is worth **61 points**.

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

1. **A Circumbinary Planet [13 pts].** A star system has two similar stars in an orbit with an apparent semimajor axis of 2", and the parallax to the system is 100 mas.
   (a) Estimate the semimajor axis of the orbit in au. [4 pts]
   (b) If the orbital period of the binary is estimated to be 100 yr, estimate the total mass in the binary and the approximate mass of each component. What kind of stars are these likely to be? [5 pts]
   (c) The binary is seen to host a circumbinary planet — i.e., a planet in a wide orbit around the two stars — at a separation from the binary of 20". Estimate the planet's orbital velocity in SI units. [4 pts]
2. **Binaries [4 pts]**. Sketch or plot the approximate distribution of orbital periods (in years) for Sun-like stars in binary systems. Indicate the orbital period of the binary described in Question 1. How does it compare to the overall distribution of binaries?
3. **Orbital Energy [10 pts]**. Assume a planet of mass m is in a circular orbit with semimajor axis a around a star of mass M_*.
   (a) What is the gravitational potential energy of this two-body binary system? [1 pt]
   (b) What is the kinetic energy of the system in terms of the planet's orbital velocity v? [1 pt]
   (c) Describe (or draw a diagram) indicating the direction and magnitude of the astronomically relevant forces acting on the planet. [2 pts]
   (d) Using Newton's Second Law, derive an expression for the planet's orbital velocity v^2 in terms of the other quantities given. [3 pts]
   (e) Write an expression for the total energy in the system in terms of m, M_*, and a but not v. [3 pts]
4. **What is a planet? [10 pts]** Aside from the general IAU definition of a 'planet,' there are several proposals for more quantitative metrics to distinguish a planet from a pla-NOT (that last one is a joke). One is

$$\Pi = k_\Pi \frac{m}{M_\oplus} \left(\frac{M_*}{M_\odot} \right)^{-5/2} \left(\frac{a}{1 \text{ au}} \right)^{-9/8}; \qquad (1)$$

   where for the Solar System $k_\Pi \approx 800$. A second, alternative metric is

$$\Lambda = k_\Lambda \left(\frac{m}{M_\oplus} \right)^2 \left(\frac{a}{1 \text{ au}} \right)^{-3/2} \qquad (2)$$

   with $k_\Lambda \approx 1.5 \times 10^5$. Under either a proposal, an object is considered a planet when the metric (Π or Λ) exceeds unity.
   (a) Plot both metrics vs. a for planets with $m = M_\oplus$ and M_{Pluto}. Discuss when such objects might be considered truly planets. [5 pts]
   (b) Plot both metrics vs. m for planets with a=1 au and a=100 au. Discuss. [5 pts]
5. **Radiation Units [6 pts]**. Define each of the following, and give the SI or 'astronomer's' units they are measured in: energy, luminosity, and flux.

6. **Stellar Classification [12 pts]**. Plot the following on a Hertzsprung-Russell diagram, with axes of $T_{\rm eff}$/K and $L/L_\odot$.

   - The Sun.
   - An A0V star with typical $R_* = 2R_\odot$.
   - A red dwarf with typical $R_* = 0.5R_\odot$.
   - The Wolf-Rayet star γ Velorum.
   - A red supergiant with $R_* = 100R_\odot$.
   - A white dwarf (dead stellar remnant) roughly the size of the Earth but 10,000 K.

7. **Order of Magnitude [6 pts]**. In SI units:

   (a) Estimate the luminosity of an ice cube.

   (b) Estimate the luminosity of your body.

   (c) Estimate the luminosity of the Earth.