UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2020

Problem Set 4
Due: Wed, May 11, 2019, in class
This problem set is worth **63 points**.

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

1. **Taking the Temperature of a Planet [14 pts].** Consider an arbitrary exoplanet with as-yet unspecified parameters. Its host star has luminosity L_*, temperature T_*, and radius R_*, while the planet has radius R_P, temperature T_P, and it's on a nearly-circular orbit with semimajor axis a.

   (a) What is the luminosity of the *planet* (not the star), in terms of the given quantities? [2 pts]

   (b) What is the flux of the star at the planet's orbit, in terms of L_* and other relevant quantities? [2 pts]

   (c) What fraction of the star's emitted energy hits the planet and is absorbed by it? (Assume the planet has zero albedo, so all light hitting it is absorbed). [3 pts]

   (d) [4 pts] Show that by equating the planet's thermal emission (luminosity) and the stellar energy it absorbs, the approximate temperature of the planet is

   $$T_P = \left(\frac{R_*}{a}\right)^{1/2} T_*. \tag{1}$$

   (e) If the star is our Sun, estimate the planetary temperature this relation gives for Venus, Earth, and Mars. [3 pts]

2. **Observing a Star [13 pts]**. As you see on your OoMA sheet, a star with $m_V = 0$ mag has a flux density (measured in photons/s, not J/s) of around 10^8 photons/s/m^2/nm at visible wavelengths.

   (a) What is the energy of a V-band photon, in Joules? [2 pts]

   (b) Use your answer in part (a) to estimate the flux density F_λ of such a star in W/m^2/nm. [3 pts]

   (c) If the V photometric band spans roughly from 500–600 nm, use your answer for F_λ to estimate the visible-wavelength flux F from the star. How does this compare to the "solar constant" flux of $\sim$1400 W m^{-2}? [4 pts]

   (d) If a second, fainter star with $m_V = 5$ were observed, by what factor would it be fainter than the first star? What would be its flux density (in photons/s/m^2/nm)? [4 pts]

3. **How Do You Say "Absorption" in Astrophysics? [12 pts]**

   (a) Define α_λ, extinction coefficient (in words, not just an equation!), and give its typical units. [3 pts]

   (b) Define σ_λ, cross-section (in words, not just an equation!), and give its typical units. [3 pts]

   (c) Define κ_λ, opacity (in words, not just an equation!), and give its typical units. [3 pts]

   (d) Define τ_λ, the optical depth (in words, not just an equation!), and give its typical units. [3 pts]

4. **A Hazy Morning [11 pts]** On a given day the weather report mentions that visibility is about one km. Take that as the distance at which $\tau_\lambda \approx 1$:

   (a) Estimate the atmosphere's extinction coefficient α_λ that morning. [3 pts]

   (b) If the visibility is mainly limited by smoke from nearby fires, estimate the effective cross-section σ_λ of the smoke particles and the number density n of these particles in the atmosphere. [4 pts]

   (c) Given the typical density ρ of air at sea level, estimate the opacity κ_λ of the smoky air. [4 pts]

5. **Basic Radiative Transfer [13 pts]**

(a) How does the specific intensity (I_λ) of light change as the light passes through empty space? [3 pts]

(b) How does I_λ change as light passes through an absorbing medium (e.g., dust, fog, etc.)? Assume the medium emits no light of its own. [3 pts]

(c) How does I_λ change as light passes through a hot, incandescent medium? Assume the medium absorbs none of the light passing through it. [3 pts]

(d) How does I_λ change as light passes through material that is hot & incandescent, but that also absorbs some of the light passing through it? [4 pts]