W15L2 – Worked Problems

Extra Credit Opportunities on May 12:
Donuts & Black Holes: 0800-1130 (MAL 3005)
Public talk and sky show: at 1930 (MAL 1001)

Final exam: Friday, May 13, **10:30 AM**

Please complete the online course evaluation
(why not do it right now, before class?)

Name:____________________________

UNIVERSITY OF KANSAS

Department of Physics and Astronomy

ASTR 391 — Prof. Crossfield — Spring 2022

Final Exam

Friday, 2022/05/13

This exam is worth **90 points**. Please complete all questions in the exam. As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures, and appropriately label all plots. Unlike a problem set for this midterm you may use only this equation sheet, the OoMA factoid sheet, and a 'dumb' calculator.

$$\frac{\text{(parallax angle)}}{1 \text{ arcsec}} = \frac{1 \text{ pc}}{\text{distance}} \tag{1}$$

$$\textbf{Stefan – Boltzmann Law}: \quad L = (\text{surface area}) \times \sigma_{SB} T^4 \qquad \frac{L_*}{L_\odot} = \left(\frac{R_*}{R_\odot}\right)^2 \left(\frac{T_*}{T_\odot}\right)^4 \tag{2}$$

$$\textbf{Wien's Law}: \quad \frac{\lambda_{max}}{1000 \text{ nm}} \approx \frac{3000 \text{ K}}{T} \tag{3}$$

$$c = \lambda\nu \qquad E_{\text{photon}} = h\nu \qquad \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c} = z - 1 \tag{4}$$

$$E_{\text{mass}} = mc^2 \tag{5}$$

$$P = nk_BT \qquad n = \frac{\rho}{\langle m_{\text{particle}} \rangle} \tag{6}$$

$$M_{\text{enc}}(r) = \int_0^r 4\pi r^2 \rho(r) dr \tag{7}$$

$$\textbf{Gravity}: \quad F_G = G\frac{m_1 m_2}{r^2} \qquad g(r) = \frac{GM_{\text{enc}}(r)}{r^2} \tag{8}$$

$$\textbf{Hydrostatic Equilibrium}: \quad \frac{dP}{dr} = -\rho(r)g(r) \qquad P(r) = \int_r^{R_*} \rho(r)g(r)dr \tag{9}$$

$$\textbf{Kepler's 3rd Law}: \quad P^2\left(\frac{GM_{tot}}{4\pi^2}\right) = a^3 \qquad \left(\frac{P}{1 \text{ yr}}\right)^2 \frac{M_{tot}}{M_\odot} = \left(\frac{a}{1 \text{ AU}}\right)^3 \tag{10}$$

$$\textbf{Mass – Luminosity Relation}: \quad L_{\text{dwarf star}} \underset{\sim}{\propto} M_*^4 \tag{11}$$

$$L = \frac{E}{\Delta t} \qquad F = \frac{L}{\text{area}} \qquad F_\lambda = \frac{F}{\Delta\lambda} \qquad I_\lambda = \frac{F_\lambda}{\text{(angular area)}} \tag{12}$$

$$E = \int L dt \qquad L = \int F\, dA \qquad F = \int F_\lambda\, d\lambda \qquad F_\lambda = \int I_\lambda\, d(\text{angle})^2 \tag{13}$$

$$\textbf{Hubble and Friedmann Equations}: \quad v = H_0 d \qquad H^2 + \frac{kS}{d^2} = \frac{8}{3}\pi G \rho_{\text{universe}} \tag{14}$$

Final Exam:

- Cumulative
- Six questions
- Equation sheet (*left*) and OoMA sheet provided
- Closed-notes
- 'Dumb' calculator allowed (trig: yes, programmable: no)

https://crossfield.ku.edu/A391_2022A/W15L2_problems.pdf

1) Angles and Orbits

A) A star has a measured parallax of 0.002 arcsec. Estimate its distance in parsecs.

B) The star is observed to have a planet orbiting it with an apparent semimajor axis of 0.1 arcsec. What is the semimajor axis of the orbit, in AU?

C) The planet's orbital period is (eventually!) observed to be 200 years. What is the mass of the star?

II) Cosmology

A) You slip into an alternate universe in which H_0 has a value 10 times greater than its value in our universe. What is the value of the alt-universe's Hubble constant, H_0, in its usual units?

B) How might you expect the history of the alt-universe differ from that of our universe?

C) Estimate the critical density, ρ_{crit}, of this alternative universe.

III) Our Neighbor, Venus

A) The orbital period of Venus is 220 days. Estimate its semimajor axis, in AU.

B) Estimate how much more intense the sun's flux is on Venus than on Earth (i.e., the ratio of the two planets' "solar constants").

C) Venus emits light most strongly in the infrared at roughly 15 μm. Estimate the temperature of Venus.

D) Venus is roughly the same size as the Earth. Estimate its luminosity relative to that of the Sun.

IV) Stellar Evolution & Energy

A) Estimate the luminosity (relative to the Sun) of a star twice the size of the sun and 50% hotter at the surface.

B) Estimate the nuclear fusion timescale for this star, compared to the Sun’s fusion timescale.

C) If this star was twice the mass of the sun while on the main sequence, roughly what fraction of its mass will it have lost by the time its life ends?

V) Stellar Interiors

A) What sequence of steps would you take to calculate the gas pressure throughout a star, P(r), given only the star's density profile $\rho(r)$?

B) Roughly sketch the density and enclosed mass profiles vs. radius for a typical star.