4 Temperature, Luminosity, and Energy

Blackbody Emission

The stellar spectra plotted in Fig. are distinct but qualitatively similar in some respects. For example, if one squints at them to blur out the details of the various spectral absorption features, all the stellar spectra start out fairly faint at short wavelengths, rise to a maximum brightness at some intermediate wavelength, and then fade again toward longer wavelengths. This behavior is characteristic of a **blackbody** emission spectrum. Stars are not perfect blackbodies (they have spectral features, after all) but they are often reasonably close.

The particular shape of a blackbody spectrum is given by the **Planck blackbody function**

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_B T} - 1} \tag{15}$$

is of critical importance in astrophysics. The Planck function tells how bright an object with temperature T is as a function of frequency ν. Note that the Planck function can also be written in terms of wavelength λ, but you can't just replace the ν's in Eq. 15 with λ's: instead one must write the identity $\lambda B_\lambda = \nu B_\nu$ and calculate $B_\lambda(T)$ from there.

It's worth plotting $B(T)$ for a range of temperatures to see how the curve behaves, as shown in Fig. 3. One interesting result is that the wavelength of maximal intensity turns out to scale linearly with T. This so-called **Wien Peak** is approximately

$$\lambda_{\max} T \approx 3000 \mu m\ K \tag{16}$$

So radiation from a human body peaks at roughly 10μm in the mid-infrared, while that from a 6000 K, roughly Sun-like star peaks at 0.5μm = 500 nm — right in the response range of the human eye.

Another important correlation is the link between a blackbody's luminosity L and its temperature T. For any specific intensity I_ν, the bolometric flux F is given by Eqs. 91 and 92. When $I_\nu = B_\nu(T)$, the **Stefan-Boltzmann Law** directly follows:

$$F = \sigma_{SB} T^4 \tag{17}$$

where σ_{SB}, the Stefan-Boltzmann constant, is

$$\sigma_{SB} = \frac{2\pi^5 k_B^4}{15 c^2 h^3} \tag{18}$$

(or $\sim 5.67 \times 10^{-8}$ W m^{-2} K^{-4}).

Assuming **isotropic** emission (i.e., that the star shines equally brightly in all directions), the luminosity of a sphere with radius R and temperature T is

$$L = 4\pi R^2 F = 4\pi\sigma_{SB} R^2 T^4 \tag{19}$$

.

If we assume that the Sun is a blackbody with $R_\odot \approx 7 \times 10^8$ m and $T \approx$ 6000 K, then we would calculate

$$L_\odot \approx 4 \times 3 \times (6 \times 10^{-8}) \times (7 \times 10^8)^2 \times (6 \times 10^3)^4 \tag{20}$$
$$= 72 \times 10^{-8} \times (50 \times 10^{16}) \times (1000 \times 10^{12}) \tag{21}$$
$$= 3600 \times 10^{23} \tag{22}$$

which is surprisingly close to the IAU definition of $L_\odot = 3.828 \times 10^{26}$ W s^{-1}.

Soon we will discuss the detailed structure of stars. Again, their spectra (Fig. 4) show that they are not perfect blackbodies, but they are often pretty close. This leads to the common definition of an **effective temperature** linked to a star's size and luminosity by the Stefan-Boltzmann law, as shown by rearranging Eq. 19 to find Eq. 13. In other words, the effective temperature is the temperature of a blackbody with the same size and luminosity of the star.

4.1 *Units of Luminosity, Flux, and Blackbody Emission*

An important note on the units of these various quantities. Luminosity is the quantity that should be most familiar to you: this is just a power (energy per time) and measured in W=J s^{-1}. Specifically, the luminosity is the total power emitted by an object integrated over all wavelengths (or frequencies), from X-rays to radio waves.

We will also often talk about **flux**, which is the amount of power passing

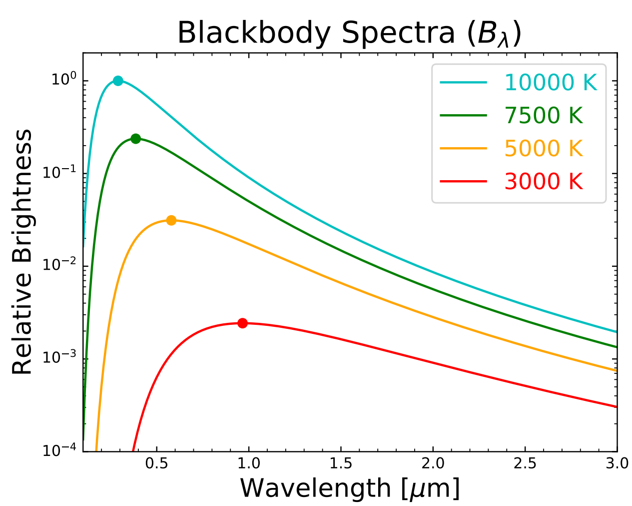

Figure 3: Blackbody spectra $B_\lambda(T)$ for a range of temperatures T. The temperatures used here correspond roughly to the range of T_{eff} spanned by the stars shown in Fig. 4. The circular points indicate Wien peak of each blackbody.

through some area. If you know the luminosity of an object, the flux we measure from it is just the power spread out over some large surface area. For a star or similar object that radiates equally in all directions, the radiation goes out spherically and so the flux at some distance r is just

$$F = \frac{L}{4\pi r^2}. \tag{23}$$

The SI units of flux are W m^{-2}. One common example is the so-called **solar constant** (the flux incident on the Earth from the Sun). This is approximately

$$F_{\oplus} = \frac{L_{\odot}}{4\pi(1 \text{ au})^2} \approx \frac{4 \times 10^{26}}{12 \times (1.5 \times 10^{11})^2} \approx \frac{10^4}{6} \text{ W m}^{-2}. \tag{24}$$

A more precise value is about 1400 W m^{-2}; this is a critical value for modeling weather, environmental behavior, and solar power generation.

As for the Planck function, it is measured in neither luminosity nor flux; instead its units are something called specific intensity that we will come back to later.