7 Binary Systems

Having dealt with the two-body problem, we'll leave the three-body problem to science fiction authors[3] and begin an in-depth study of stars. Our foray into Kepler's laws and two-body systems was appropriate, because about 50% of all stars are in binary (or higher-multiplicity) systems. With our fundamental dynamical model, plus data, we get a lot of stellar information from **binary stars** — that is, stars in two-star systems.

Note that there are increasing levels of stellar multiplicity: systems of three, four, or even more stars all orbiting each other in a complicated dance. These higher-order multiples are less common than binaries, and they are often arranged heirarchically: that is, a triple system will typically involve two stars closely-orbiting each other with the third at a much wider separation. The nearby α Centauri triple system (our closest stellar neighbors) is such a **heirarchical multiple**: the more massive stars α Cen A and α Cen B orbit each other with $a = 23$ au, while the smaller, cooler, lower-mass Proxima Centauri (the closest star to the Sun) orbits A & B at roughly $a \approx 8000$ au.

Stars in binaries are best characterized by mass M, radius R, and luminosity L. Note that an effective temperature $T_{\rm eff}$ is often used in place of L (see Eq. 13). An alternative set of parameters from the perspective of stellar evolution would be M; heavy-element enhancement metallicity [Fe/H], reported logarithmically; and age.

7.1 *Empirical Facts about binaries*

The distribution of stellar systems between singles, binaries, and higher-order multiples is roughly 55%, 35%, and 10% (for details see Raghavan et al. 2010[4]) – so the average number of stars per system is something like 1.6.

Orbital periods range from < 1day to $\sim 10^{10}$ days ($\sim 3 \times 10^6$ yr). Any longer, and Galactic tides will disrupt the stable orbit (the Sun takes ~ 200 Myr to orbit the Milky Way). The periods of binary stars have a **log-normal distribution** – that is, the distribution is roughly Gaussian in $\log(P)$. For Sun-like stars, binaries are most common with $\log_{10}(P/d) = 4.8$ with a width of 2.3 dex (Duquennoy & Mayor 1992[5]) – that is, the most common orbital periods are roughly $10^{4.8 \pm 2.3}$ d.

There's also a wide range of eccentricities, from nearly circular to highly elliptical. For short periods, we see $e \approx 0$. This is due to tidal circularization: the stretching and squeezing of a 3D (non-pointlike) body by a companion's gravity 'steals' energy from the orbit, causing eccentric orbits to eventually circularize. Stars and planets aren't point-masses and aren't perfect spheres; tides represent the differential gradient of gravity across a physical object, and they bleed off orbital energy while conserving angular momentum. It turns out that this means e decreases as a consequence, as explained by the energy diagram analysis presented in Sec. 6.4.

[3] See: `https://en.wikipedia.org/wiki/The_Three-Body_Problem_(novel)` .
[4] `https://ui.adsabs.harvard.edu/abs/2010ApJS..190....1R/`
[5] `https://ui.adsabs.harvard.edu/abs/1991A%26A...248..485D/`

7.2 *Parameterization of Binary Orbits*

As discussed before in Sec. 6.2, two bodies orbiting in 3D requires 12 parameters, three for each body's position and velocity. Three of these map to the 3D position of the center of mass – we get these if we measure the binary's position on the sky and the distance to it. Three more map to the 3D velocity of the center of mass – we get these if we can track the motion of the binary through the Galaxy.

So we can translate any binary's motion into its center-of-mass rest frame, and we're left with **six numbers describing orbits** (see Fig. 9):

- P – the orbital period
- a – semimajor axis
- e – orbital eccentricity
- I – orbital inclination relative to the plane of the sky
- Ω – the longitude of the ascending node
- ω – the argument of pericenter

The period gives the relevant timescale; the next two parameters give us the shape of the ellipse; the last three describe the ellipse's orientation (three angles for 3D space, as you may have seen in classical mechanics).

7.3 *Binary Observations*

The best way to measure L comes from basic telescopic observations of the apparent bolometric flux F (i.e., integrated over all wavelengths). Then we have

$$F = \frac{L}{4\pi d^2} \quad (55)$$

where ideally d is known from parallax.

But the most precise way to measure M and R almost always involve stellar binaries (though asteroseismology can do very well, too). But if we can observe enough parameters to reveal the Keplerian orbit, we can get masses (and separation); if the stars also undergo eclipses, we also get sizes.

In general, how does this work? We have two stars with masses $m_1 > m_2$ orbiting their common center of mass on elliptical orbits. Kepler's third law says that

$$\frac{GM}{a^3} = \left(\frac{2\pi}{P}\right)^2 \quad (56)$$

so if we can measure P and a we can get M. For any type of binary, we usually want $P \lesssim 10^4$ days if we're going to track the orbit in one astronomer's career.

If the binary is nearby and we can directly see the elliptical motion of at least one component, then we have an **astrometric binary**. If we know the

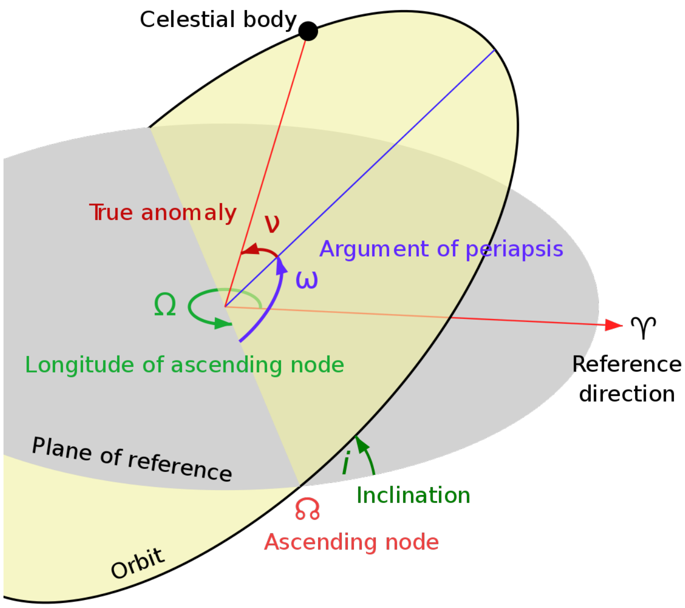

Figure 8: Geometry of an orbit. The observer is looking down along the z axis, so x and y point in the plane of the sky.

distance d, we can then directly determine a as well (or both a_1 and a_2 if we see both components). The first known astrometric binary was the bright, northern star Sirius – from its motion on the sky, astronomers first identified its tiny, faint, but massive white dwarf companion, Sirius b.

More often, two objects in a binary are so close that we can't separate the light well enough to see their astrometric motion. In these cases, we obtain spectra of the stars that let us measure the stars' Doppler shifts, and so measure the velocity of one or both stars. If we can only measure the periodic velocity shifts of one star (e.g. the other is too faint), then the **spectroscopic binary** is an "SB1". If we can measure the Doppler shifts of both stars, then we have an "SB2": we get the individual semimajor axes a_1 and a_2 of both components, and we can get the individual masses from $m_1 a_1 = m_2 a_2$.

If we have an SB1, we measure the radial velocity of the visible star. Assuming a circular orbit,

$$v_{r1} = \frac{2\pi a_1 \sin I}{P} \cos\left(\frac{2\pi t}{P}\right) \quad (57)$$

where P and v_{r1} are the observed quantities. What good is $a_1 \sin I$? We know that $a_1 = (m_2/M)a$, so from Kepler's Third Law we see that

$$(58)\quad \left(\frac{2\pi}{P}\right)^2 = \frac{Gm_2^3}{a_1^3 M^2}$$

Combining Eqs. 57 and 58, and throwing in an extra factor of $\sin^3 I$ to each side, we find

$$(59)\quad \frac{1}{G}\left(\frac{2\pi}{P}\right)^2 a_a^3 \sin^3 I = \frac{1}{G}\frac{v_{r1}^3}{(2\pi/P)}$$

$$(60)\quad = \frac{m_2^3 \sin^3 I}{M^2}$$

where this last term is the spectroscopic "mass function" – a single number built from observables that constrains the masses involved.

$$(61)\quad f_m = \frac{m_2^3 \sin^3 I}{(m_1+m_2)^2}$$

In the limit that $m_1 << m_2$ (e.g. a low-mass star or planet orbiting a more massive star), then we have

$$(62)\quad f_m \approx m_2 \sin^3 I \le m_2$$

Another way of writing this out in terms of the observed radial velocity semi-amplitude K (see Lovis & Fischer 2010) is:

$$(63)\quad K = \frac{28.4 \text{ m s}^{-1}}{(1-e^2)^{1/2}} \frac{m_2 \sin I}{M_{Jup}} \left(\frac{m_1+m_2}{M_\odot}\right)^{-2/3} \left(\frac{P}{1 \text{ yr}}\right)^{-1/3}$$

Fig. 9 shows the situation if the stars are eclipsing. In this example one star

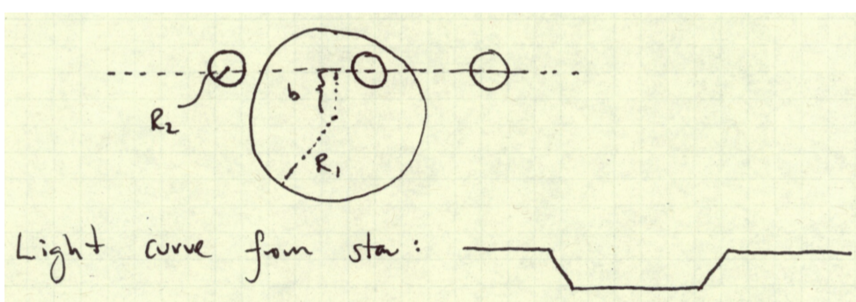

Figure 9: Geometry of an eclipse (top), and the observed light curve (bottom).

is substantially larger than the other; as the sizes become roughly equal (or as the impact parameter b reaches the edge of the eclipsed star), the transit looks less flat-bottomed and more and more V-shaped.

If the orbits are roughly circular then the duration of the eclipse (T_{14}) relates directly to the system geometry:

$$T_{14} \approx \frac{2R_1\sqrt{1-(b/R_1)^2}}{v_2} \tag{64}$$

while the fractional change in flux when one star blocks the other just scales as the fractional area, $(R_2/R_1)^2$.

There are a lot of details to be modeled here: the proper shape of the light curve, a way to fit for the orbit's eccentricity and orientation, also including the flux contribution during eclipse from the secondary star. Many of these details are simplified when considering extrasolar planets that transit their host stars: most of these have roughly circular orbits, and the planets contribute negligible flux relative to the host star.

Eclipses and spectroscopy together are very powerful: visible eclipses typically mean $I \approx 90^o$, so the $\sin I$ degeneracy in the mass function drops out and gives us an absolute mass. Less common is astrometry and spectroscopy – the former also determines I; this is likely to become much more common in the final Gaia data release (DR4, est. 2022).