UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2022

Problem Set 5
Due: Monday, Mar 21, 2021, in class
This problem set is worth **30 points** (plus 10 potential bonus points).

As always, be sure to: type and print your solutions, show your work, circle your final answer, and use the appropriate number of significant figures.

1. **Journey to the Center of a Star [30 pts]**

   (a) [5 pts] In lecture we discussed at some length how we can model the physical conditions in a star's interior. Assume the star has a slightly more realistic density profile (valid from $0 \leq r \leq R_*$) of

   $$\rho(r) = \rho_c \left((r/R_*)^2 - 2r/R_* + 1 \right) = \rho_c \left(\frac{r}{R_*} - 1 \right)^2 \tag{1}$$

   Plot $\rho(r)$ over the full range from $r = 0$ to $r = 2R_*$. Discuss why this density profile might be slightly more realistic than the constant-density model we assumed in class.

   (b) [5 pts] Show that the expression for the Enclosed Mass $M_{enc}(r)$ at an arbitrary radius r within this star is

   $$4\pi\rho_c r^3 \left(\frac{r^2}{5R_*^2} - \frac{r}{2R_*} + \frac{1}{3} \right) \tag{2}$$

   (c) [5 pts] Using the expression above for $M_{enc}(r)$, show that the gravitational acceleration $g_{in}(r)$ inside this star will be

   $$g(r) = 4\pi\rho_c G r \left(\frac{r^2}{5R_*^2} - \frac{r}{2R_*} + \frac{1}{3} \right). \tag{3}$$

   .

   (d) [6 pts] Plot $M_{enc}(r)$ and $g_{in}(r)$ over the range from $r = 0$ to $2R_*$.

   (e) [4 pts] Starting with the equation of hydrostatic equilibrium ($dP/dr = -\rho(r)g(r)$), we could continue in this vein and calculate the internal pressure and temperature of the star – but things would quickly get really messy.

   Instead, we can make a rough approximation to get a sense of the conditions inside the star, by assuming that $dP/dR \approx P_c/R_*$ (i.e., the pressure at the center of the star divided by the star's radius), and further assuming density and gravity are constant: $\rho(r) = \rho_{\rm avg}$ and $g(r) = g_{\rm surface}$.

   Under these simplifying assumptions, the pressure at the center of the star is just $P_c \approx \rho_{\rm avg} g_{\rm surf} R_*$. Calculate a numerical value of P_c (in SI units) for the Sun, and for a red dwarf with $M_*/M_\odot = R_*/R_\odot = 0.3$. How do these compare to the atmospheric pressure here on Earth?

   (f) [5 pts] Assume that our star is an ideal gas made entirely of hydrogen atoms. In this case, derive a symbolic expression for the temperature T_c at the center of a star in terms of its pressure and density.

   Then, calculate a numerical value for the central temperature of both the Sun and the M dwarf described above. How do these compare to the surface temperatures of these stars?

2. BONUS: Under Pressure [10 pts]

   (a) [6 pts] Using Equations 1, 2, and 3, calculate an analytic expression (i.e., a formula) for the pressure inside the star.

   (b) [4 pts] Plot $P(r)$ over the range from $r = 0$ to $2R_*$.