UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2022

Problem Set 7
Due: Monday, April 25, 20222, 11am Kansas Time
This problem set is worth **53 points**.

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

1. **Our home, the Milky Way [15 pts]** Draw a rough sketch of our Milky Way galaxy as it might be viewed from an outside observer, as both a top-down and edge-on view. At a minimum, include and label the Galactic Center, bulge, halo, disk, spiral arms, and the Sun, as well as a scale bar.

2. **Other Galaxies [15 pts]**

   (a) [3 pts] List three ways astronomers might measure the distance to a distant Galaxy.

   (b) [3 pts] A 2019 article in the journal *Nature* celebrated that year's Nobel Prize by writing that the award was given for "the first report of an exoplanet orbiting around a Sun-like star in another galaxy." Describe what is wrong with this statement, and how you would correct it.

   (c) [3 pts] The closest sizable galaxy to the Milky Way is Andromeda (also known as M31), which you can see from the Northern hemisphere on a dark night in the Autumn. If Andromeda is 770 kpc away, and is roughly the same physical size as the Milky Way, what is its approximate angular diameter on the sky?

   (d) [6 pts] Sketch the 'tuning-fork' diagram used for galaxy classification, and describe the properties of the main types of galaxies shown there.

3. **Space is Big. Really Big. [10 pts]** Plot the distance to the following objects on a (1D) logarithmic number line (e.g., with regularly-spaced intervals on the page indicating 1/10, 1, 10×, etc.):

   (a) Moon

   (b) Sun (closest star)

   (c) Neptune

   (d) Alpha Centauri (closest non-Sun star)

   (e) Sagittarius A*

   (f) Large Magellanic Cloud

   (g) Andromeda Galaxy

   (h) Distant galaxy with redshift $z = 1$ ($d = 4.3$ Gpc).

4. **Rotation Curves [13 pts]** A key insight in astronomy is that if we can see one object orbiting another, we can measure the total mass enclosed by the orbit using Kepler's Third Law. In Solar-System units, this is

$$\left(\frac{P}{\mathrm{yr}}\right)^2 \frac{M_{\mathrm{enclosed}}}{M_\odot} = \left(\frac{r}{\mathrm{AU}}\right)^3. \tag{1}$$

   This applies to binary asteroids, planets with moons, stars with planets, and – as you will see here – even for distant galaxies.

   (a) [5 pts] Using the expression above, show that for a small mass in a circular orbit a distance r from a much larger amount of mass M_{enclosed}, the orbital speed just depends on the mass interior to the orbit as

$$v_{\mathrm{orb}} \approx 30 \text{ km s}^{-1} \sqrt{\frac{M_{\mathrm{enc}}/M_\odot}{r/\mathrm{AU}}}. \tag{2}$$

   (As an alternative approach, you can try equating the gravitational force between the objects to the expression for a centripetal force.)

(b) [3 pts] A wide range of objects orbit our Sun, from Mercury out to Neptune, Pluto, and beyond. Assuming all objects orbiting the Sun have circular orbits, plot their orbital speeds vs. semimajor axis — i.e., plot the above expression v_{orb}, the rotation curve for things orbiting around a single massive object. (Note that this should not be a hand-drawn sketch, but rather a computer-generated graph.)

(c) [5 pts] Roughly sketch the rotation curve (v_{orb} vs. r) for a typical spiral galaxy. How is this similar to the previous plot (for the Solar System)? How is it different? What do the differences imply for what makes up most of the mass in galaxies, vs. what makes up most of the mass in a Solar system?

5. [13 pts] **BONUS POINTS** (optional): Assume you have a spherically symmetric cloud of material, with a density profile $\rho(r) = \ell_0 r^{-2}$, where ℓ_0 is some reference density with units kg/m. (Obviously this functional form is nonphysical since $\rho(r=0) \to \infty$, but set that aside for now – it's only a model).

   (a) Calculate M_{enclosed} as a function of r and ℓ_0 (as necessary). [3 pts]

   (b) Use M_{enclosed} to calculate the orbital velocity v_{orb} for objects orbiting in this cloud as a function of r and ℓ_0 (as necessary). [4 pts]

   (c) If this object is a galaxy and the rotation speed of stars in its outer reaches ($r_{\mathrm{max}} \approx 10$ kpc) is observed to be roughly 200 km/s, what is ℓ_0 (in kg/m)? Then (the final goal!) use ℓ_0 to calculate the total mass involved, $M_{\mathrm{enc}}(r = r_{\mathrm{max}})$, in Solar masses $M_\odot$. [6 pts]