18 An Introduction to Nuclear Fusion

18.1 *Useful References*

- Choudhuri, Secs. 4.1–4.2
- Kippenhahn, Weiger, and Weiss, 2nd ed., Chap. 18
- Hansen, Kawaler, and Trimble, Sec. 6.2

18.2 *Introduction*

Commercial nuclear fusion may be perpetually 50 years away, but stellar fusion has powered the universe for billions of years and (for the lowest-mass stars) will continue to do so for trillions of years to come.

Our two goals here are (1) to understand ϵ, the volumetric energy production rate (see Eq. 289), and how it depends on ρ and T; and (2) to identify and describe the key nuclear reaction pathways that are important in stars.

18.3 *Nuclear Binding Energies*

Stars derive their energy from the fusion of individual atomic nuclei, as we described briefly in Sec. 14.6. Fusion involves true elemental transmutation of the sort that the ancients could only dream of. For better or for worse, our own discussions of this natural alchemy will involve relatively more considerations of the detailed physics involved and relatively less boiling of one's own urine.

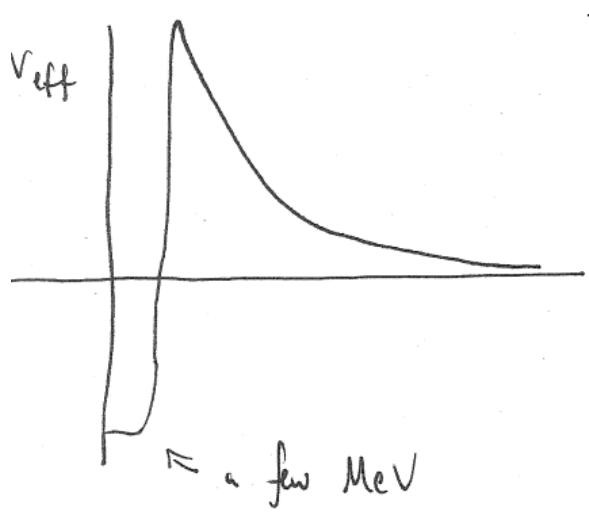

Figure 33: Rough sketch of the nuclear potential. Coulomb (electrostatic) repulsion dominates at large separations, and is overwhelmed by Strong nuclear attraction at the smallest separations.

Nuclei all have positive charges, so they're generally inclined to repel each other rather than to fuse. For one nucleus to reach another and fuse, it must overcome the strong Coulomb (electrostatic) repulsion generated by the two positively-charged nuclei. Fig. 33 gives a rough sketch of the situation: Coulomb repulsion dominates at large separations, but it is overwhelmed by the Strong nuclear force, which is attractive at the smallest separations.

The fundamental nuclear size is set by the typical radius of protons and neutrons, $r_p \approx r_n \approx 0.8$ fm, as

$$r_{\text{nuc}} \approx 2 r_p A^{1/3} \tag{376}$$

where A is the number of **nucleons** (neutrons plus protons, also approximately the atomic weight). We will also deal shortly with Z, the **nuclear charge** (i.e., number of protons). For our purposes here, all the negatively-charged electrons are so far away from the nucleus that they might as well not be there at all. (Also, stellar cores are hot enough that lighter elements are often fully ionized, i.e. all their electrons have gone and left the nucleus all alone.)

The key thing that matters in nuclear reactions is the **nuclear binding energy**. Just like two stars in a binary system are bound together via their gravitational potential energy. What's different now is that the masses involved are much smaller (protons vs. stars!), so small that we have to pay close attention to mass-energy equivalence – that old equation

$$E = mc^2. \tag{377}$$

The SI units of energy is of course the Joule, but in nuclear reactions the energies involved are much smaller than 1 J so we often use the units of eV, where 1 J$\approx 6 \times 10^{18}$ eV.

Through the equation $E = mc^2$, we will often speak of the "mass" of small particles in units of energy. So when we say an electron has a mass (more accurately, a mass energy) of ≈500 keV (that is, 500,000 eV) we just mean that $m_e c^2 \approx 500$ keV. Similarly, a proton has a mass energy of ≈1,000 MeV — and so is about 2000× more massive than an electron.

For astrophysical purposes we don't need to descend all the way into the realm of detailed nuclear physics. For our purposes an empirically-calibrated, semiclassical model (the "Bethe-Weizsäcker formula) is sufficiently accurate. This posits that the binding energy E_B of an atomic nucleus

$$E_B \approx a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} \tag{378}$$

Each of the terms in Eq. 377 has a particular significance. These are:

a_V	≈ 14 MeV	Volumetric term, describes bulk assembly of the nucleus.
a_S	≈ 13.1 MeV	Surface term, since surface nucleons have few neighbors.
a_C	≈ 0.58 MeV	Coulomb term, describes mutual repulsion of protons.
a_A	≈ 19.4 MeV	Asymmetry term, preferring $N_n = N_p$ (Fermi exclusion).

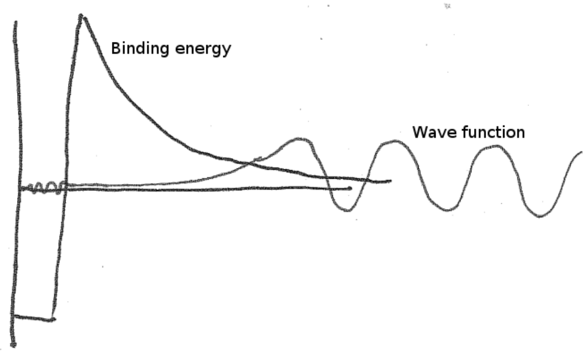

Figure 34: Rough sketch of quantum tunneling, with the wave function (just barely) penetrating the forbidding Coulomb barrier.

This model does a decent job: Eq. 377 correctly demonstrates that the nuclei with the greatest binding energy *per nucleon* have $Z \sim 25$. In fact the most tightly-bound, and thus most stable, nucleus is that of iron (Fe) with $Z = 26$, $A = 56$. Thus elements near Fe represent an equilibrium state toward which all nuclear processes will try to direct heavier or lighter atoms. For example, we will see that lighter atoms (from H on up) typically fuse into elements as high as Fe but no higher (except in unusual circumstances).

18.4 *Let's Get Fusing*

The Big Bang produced a universe whose baryonic matter was made of roughly 75% H and 25% He, with only trace amounts of heavier elements. Stellar fusion created most of the heavier elements, with supernovae doing the rest. For fusion to proceed, something must occur to either fuse H or He. Since He will have a $4\times$ greater Coulomb barrier (two nuclei with two protons each, $2 \times 2 = 4$), we'll focus on H; nonetheless we immediately encounter two huge problems.

The first big challenge is the huge Coulomb barrier shown in Fig. 33. At the separation of individual nucleons, the electronic (or protonic) repulsion is $e^2/\text{fm} \sim 1$ MeV, of roughly comparable scale to the strong nuclear attraction at shorter scales. But how to breach this Coulomb wall? Even at the center of the Sun where $T_c = 1.5 \times 10^7$ K (Sec. 14) the typical thermal energies per particle are of order $k_B T_C \sim 1$ keV — a thousand times too low. The second problem is that the fusion product of two protons would be ^{2}He, an isotope so unstable it is not entirely clear whether it has ever been observed.

Problem one: quantum tunneling

The first problem was solved by recognizing that at the nuclear scale one doesn't climb a mountain — rather, one tunnels through it. Quantum mechanics states that each particle has a wave function $\Psi(x)$ given by the Schrödinger

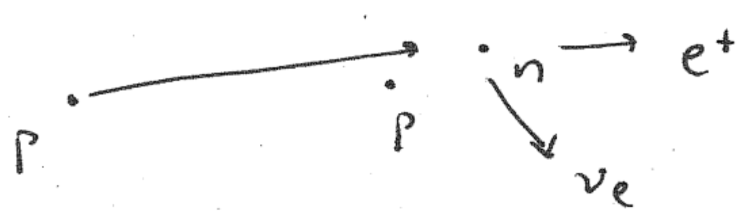

Figure 35: Rough sketch of inverse beta decay: $p + p$ yields $p + n + e^+ + \nu_e$.

Equation, and the probability of finding the particle at x is $\propto |\Psi(x)|^2$. When the particle's energy is less than required to classically overcome an energy barrier, the wavefunction decays exponentially but remains nonzero. To order of magnitude, the protons only need to get close enough to each other that their thermal de Broglie wavelengths overlap; when this happens, tunneling becomes plausible (as sketched in Fig. 34).

Problem two: avoiding the ^{2}He trap

The second challenge to fusion is that the product of H + H, ^{2}He, is incredibly unstable, and its solution lies in the humble neutron. Given sufficient neutrons we could form the stable isotope ^{2}H (deuterium) instead of ^{2}He and open up new reaction pathways.

The challenge is that the neutron half-life is only $\sim$15 min, after which neutrons undergo beta decay via $n \rightarrow p + e^- + \bar{\nu}_e$. (Here n is a neutron, p is a proton that is the nucleus of a hydrogren atom, e is an electron, and ν indicates that a neutrino is also created). The solution to our second problem lies in a related nuclear reaction, inverse beta decay. In this process (sketched in Fig. 35) two of the many, common protons interact via the Weak nuclear force. The full reaction is

$p + p \rightarrow p + n + e^+ + \nu_e \rightarrow {}^2\mathrm{H} + e^+ + \nu_e$

and perhaps surprisingly, this can provide all the neutrons we need to produce sufficient ^{2}H to make the universe an interesting place to be. The cross-section is tiny (it's a weak process):

$$\sigma_{\mathrm{p-p}} \approx 10^{-22} \text{ barnes} = 10^{-46} \text{ cm}^2 \tag{379}$$

(recall that the electron scattering, or Thomson, cross section of Sec. 13.5 was $\sigma_T = 0.67$ barns!). Put another way, the reaction rate in the Sun will be just once per $\sim$few Gyr, per proton. But it's enough, and once we have ^{2}H we can start producing heavier (and more stable) He isotopes: fusion becomes energetically feasible. Thus the solution to the ^{2}He fusion barrier is similar in a way to the H^- story of Sec. 13.5: to get everything right required both hydrogen and some imagination.