UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2024

Problem Set 5 – REVISED
Due: Friday, March 29, 2024, in class
This problem set is worth **23 points** (plus 7 bonus points).

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures. Also, please **submit your PSet as a single PDF file** (not individual scanned images, which are tougher to keep track of), and **include your name in the PDF's filename**.

Journey to the Center of a Star

For this problem, you have two options: use a computer to perform numerical integration (via programming language or, as shown in lecture, a spreadsheet), or alternatively use mathematical integration (calculus). If you use numerical integration, be sure to use >20 layers so your results will be reasonably accurate. Regardless: remembering when making plots for the question below that any good plot has labeled axes!

1. [5 pts] In lecture we discussed at some length how we can model the physical conditions in a star's interior. Assume the star has a slightly more realistic density profile (valid from $0 \leq r \leq R_*$) of

$$\rho(r) = \rho_c \left((r/R_*)^2 - 2r/R_* + 1 \right) = \rho_c \left(\frac{r}{R_*} - 1 \right)^2 \tag{1}$$

   Plot $\rho(r)$ over the full range from $r = 0$ to $r = 2R_*$. Discuss why this density profile might be slightly more reaslistic than the constant-density model we assumed in class.

2. [5 pts] Show that the Enclosed Mass profile $M_{enc}(r)$ within this star is equivalent to the expression

$$4\pi\rho_c r^3 \left(\frac{r^2}{5R_*^2} - \frac{r}{2R_*} + \frac{1}{3} \right). \tag{2}$$

   Then plot $M_{enc}(r)$.

3. [5 pts] Calculate the gravitational acceleration profile $g(r)$ inside the star, using the expression above for $M_{enc}(r)$. Then plot $g(r)$.

4. [3 pts] Starting with the equation of hydrostatic equilibrium ($dP/dr = -\rho(r)g(r)$), we could continue in this vein and calculate the internal pressure and temperature of the star – but things would quickly get really messy.

   Instead, we can make a rough approximation to get a sense of the conditions inside the star, by assuming that $dP/dR \approx P_c/R_*$ (i.e., the pressure at the center of the star divided by the star's radius), and further assuming density and gravity are constant: $\rho(r) = \rho_{\rm avg}$ and $g(r) = g_{\rm surface}$.

   Under these simplifying assumptions, the pressure at the center of the star is just $P_c \approx \rho_{\rm avg} g_{\rm surf} R_*$. Calculate a numerical value of P_c (in SI units) for the Sun, and for a red dwarf with $M_*/M_\odot = R_*/R_\odot = 0.3$. How do these compare to the atmospheric pressure here on Earth?

5. [5 pts] Assume that our star is an ideal gas made entirely of hydrogen atoms. In this case, derive a symbolic expression for the temperature T_c at the center of a star in terms of its pressure and mass density.

   Then, calculate a numerical value for the central temperature of both the Sun and the M dwarf described above. How do these compare to the surface temperatures of these stars?

6. BONUS [7 pts]: Use the equation of hydrostatic equilibrium to calculate the full pressure profile, $P(r)$, throughout the star described in parts 1–3 above. Plot $P(r)$.