UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2024

Problem Set 8
Due: Monday, April 22, 2024, 10am Kansas Time
This problem set is worth **45 points**.

As always, be sure to: show your work, circle your final answer, use the appropriate number of significant figures, label your plots and sketches, and turn in a physical copy of the final problem set.

1. **Other Galaxies [22 pts]**
   (a) [16 pts] Draw a rough sketch and give a named example of the four main types of galaxies.
   (b) [6 pts] Sketch the 'tuning-fork' diagram used for galaxy classification, and describe the properties of the main types of galaxies shown there.

2. **Space is Big. Really Big. [10 pts]** Plot the distance to the following objects on a (1D) logarithmic number line (e.g., with regularly-spaced intervals on the page indicating 1/10, 1, 10×, etc.):
   (a) Moon
   (b) Sun (closest star)
   (c) Neptune
   (d) Alpha Centauri (closest non-Sun star)
   (e) Sagittarius A*
   (f) Large Magellanic Cloud
   (g) Andromeda Galaxy
   (h) Distant galaxy with redshift $z = 1$ ($d = 4.3$ Gpc).

3. **Rotation Curves [13 pts]** A key insight in astronomy is that if we can see one object orbiting another, we can measure the total mass enclosed by the orbit using Kepler's Third Law. In Solar-System units, this is

$$\left(\frac{P}{\mathrm{yr}}\right)^2 \frac{M_{\mathrm{enclosed}}}{M_\odot} = \left(\frac{r}{\mathrm{AU}}\right)^3 . \quad (1)$$

   This applies to binary asteroids, planets with moons, stars with planets, and – as you will see here – even for distant galaxies.

   (a) [5 pts] Using the expression above, show that for a small mass in a circular orbit a distance r from a much larger amount of mass M_{enclosed}, the orbital speed just depends on the mass interior to the orbit as

$$v_{\mathrm{orb}} \approx 30 \text{ km s}^{-1} \sqrt{\frac{M_{\mathrm{enc}}/M_\odot}{r/\mathrm{AU}}} . \quad (2)$$

   (As an alternative approach, you can try equating the gravitational force between the objects to the expression for a centripetal force.)

   (b) [3 pts] A wide range of objects orbit our Sun, from Mercury out to Neptune, Pluto, and beyond. Assuming all objects orbiting the Sun have circular orbits, plot their orbital speeds vs. semimajor axis — i.e., plot the above expression v_{orb}, the rotation curve for things orbiting around a single massive object. (Note that this should not be a hand-drawn sketch, but rather a computer-generated graph.)

(c) [5 pts] Roughly sketch the rotation curve ($v_{\rm orb}$ vs. r) for a typical spiral galaxy. How is this similar to the previous plot (for the Solar System)? How is it different? What do the differences imply for what makes up most of the mass in galaxies, vs. what makes up most of the mass in a Solar system?

4. [13 pts] **BONUS POINTS** (optional): Assume you have a spherically symmetric cloud of material, with a density profile $\rho(r) = \ell_0 r^{-2}$, where ℓ_0 is some reference density with units kg/m. (Obviously this functional form is nonphysical since $\rho(r=0) \to \infty$, but set that aside for now – it's only a model).

   (a) Calculate $M_{\rm enclosed}$ as a function of r and ℓ_0 (as necessary). [3 pts]

   (b) Use $M_{\rm enclosed}$ to calculate the orbital velocity $v_{\rm orb}$ for objects orbiting in this cloud as a function of r and ℓ_0 (as necessary). [4 pts]

   (c) If this object is a galaxy and the rotation speed of stars in its outer reaches ($r_{\rm max} \approx 10$ kpc) is observed to be roughly 200 km/s, what is ℓ_0 (in kg/m)? Then (the final goal!) use ℓ_0 to calculate the total mass involved, $M_{\rm enc}(r = r_{\rm max})$, in Solar masses $M_\odot$. [6 pts]