UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2022

Problem Set 3
Due: Wednesday, February 11, 2022, by the start of class
This problem set is worth **42 points**.

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

1. **Angles, Distance, and Magnitudes [22 pts].**

   (a) Explain why an ordinary lightbulb can appear much brighter than a star, even though the lightbulb emits far less light. [3 pts]

   Solution: This is mainly because of the inverse-square law of isotropically-propogating radiation. If a light source is viewed from $3\times$ farther away, the radiation that earlier had illuminated a 1-unit-radius sphere now has to illuminate a sphere with $3^2 = 9\times$ greater surface area – so the light source will look $9\times$ dimmer.

   (b) Astronomers have measured the parallax to the stars Polaris and γ Vel ("gamma Vel," a young, hot, massive star) to be about 7.5 mas (milli-arcsec) and 2.9 mas, respectively. Estimate the distance to each star. [3 pts]

   Solution: Via the definition of parallax, $\theta/\text{arcsec} = 1\ \text{au}/(d/\text{pc})$. So

$$\boxed{d_P = (1000/7.5)\ \text{pc} \approx 130\ \text{pc}} \tag{1}$$

   and

$$\boxed{d_\gamma = (1000/2.9)\ \text{pc} \approx 340\ \text{pc}}, \text{respectively.} \tag{2}$$

   (c) In the old (pre-*Gaia*) Hipparcos astrometric catalog, the uncertainty on measured parallax was about ± 0.5 mas; roughly what distance uncertainty does this translate into for Polaris and γ Vel? (I.e.: if the parallax to Polaris is 7.5 ± 0.5 mas, what is the uncertainty range on the inferred distance?)[4 pts]

   Solution: You could solve this several ways: calculate the analytic propogation of errors for the parallax-distance equation, or calculate the distance for each value $d \pm \sigma_d$, or just conduct a quick Monte-Carlo calculation. To do the latter in Python, we could do

```
import numpy as np
n_trials = 10000 # number of Monte Carlo draws
u_p = 0.5
p_polaris_montecarlo = np.random.normal(7.5, u_p, n_trials)
p_gammavel_montecarlo = np.random.normal(2.9, u_p, n_trials)
d_polaris_montecarlo = 1000. / p_polaris_montecarlo
d_gammavel_montecarlo = 1000. / p_gammavel_montecarlo
print('Polaris: ', np.median(d_polaris_montecarlo),
    np.std(d_polaris_montecarlo))
print('Gamma Vel:', np.median(d_gammavel_montecarlo),
    np.std(d_gammavel_montecarlo))
```

   Which gives $\boxed{\sigma_\theta \approx 9\ \text{pc}}$ for Polaris and $\boxed{\sigma_\theta \approx 68\ \text{pc}}$ for γ Vel. All else being equal (e.g. equal-brightness targets), we always know the distances more precisely to things closer to us.

   (d) Describe how you might estimate the distance to a star whose parallax is too small to measure. [6 pts]

   Solution: There are several ways to do this. One would be to obtain and examine a sufficiently detailed spectrum of the star. If you can be sure that the spectrum is very similar to that of other (closer) stars whose properties (luminosity and distance) are well-known, you could use those values and the distant star's apparent magnitude to estimate the distance.

A related way would be to eschew spectra and examine broadband photometric measurements of the star's brightness. From these you could estimate the star's approximate spectral type, which would give you its $T_{\rm eff}$. If you also have a way to estimate its radius, then you can calculate its luminosity, and by comparing its absolute magnitude to its apparent magnitude you could again estimate the distance.

(e) Explain why most of the stars you can see with your own eyes in the night sky are giants and supergiants (10s to 100s of $R_\odot$), even though these stars account for only $\sim 1\%$ of all stars (most stars are $< 1 R_\odot$). [6 pts]

Solution: Giants and supergiants are larger than normal stars, so even though they are (sometimes) rather cool they still benefit from the R_*^2 term in the Stefan-Boltzmann luminosity equation; so they are far brighter than the smaller 'dwarf' stars. The sort of observational effect described in this problem is a well-known bias in observational astronomy: things that are brighter are always easier to see and count, so they seem more abundant than they really are. (If you look down on a city at night from an airplane or high-resolution satellite image, you would count more street lamps than standard-size light bulbs – but that might not be an accurate estimate of the relative number of these two types of lights in the city).

2. **Order-of-Magnitude Estimation [20 pts].**

(a) You observe a giant star that is twice the size of the Sun but has the same effective temperature. Estimate the star's luminosity in $L_\odot$.

Solution: Since we know that

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T_{\rm eff}}{T_{{\rm eff},\odot}}\right)^4 \tag{3}$$

and we have $R = 2R_\odot$ and the Solar $T_{\rm eff}$, we must have $\boxed{L = 4L_\odot}$.

(b) You observe a star that is half the size of the Sun but just 2% as luminous. Estimate the star's approximate $T_{\rm eff}$.

Solution: Again, the same relation applies. In this case we have

$$\frac{T_{\rm eff}}{T_{{\rm eff},\odot}} = \left(0.02 \times 0.5^{-2}\right)^{1/4} \approx 0.08^{1/4} \approx 0.53. \tag{4}$$

Since the Sun's effective temperature is roughly 5800 K, our star has $\boxed{T_{\rm eff} \approx 3100 \text{ K}}$.

(c) You observe a hot star that is just as luminous as the Sun but $10\times$ hotter. Estimate the star's approximate size in $R_\odot$ and in $R_\oplus$.

Solution: We can use the same relation as above, rearranged to find R. In terms of Solar units, we have

$$R = \left(10^{-4}\right)^{1/2} \to \boxed{R = 0.01 R_\odot}. \tag{5}$$

Since the Sun is roughly $100\times$ larger than the Earth, this means that our unusual hot star is tiny, 'just' the size of a planet: $\boxed{R \approx R_\oplus}$.

(d) Estimate the wavelengths at which each of the three of the stars above emit most of their light. [4 pts]

Solution: We need to use Wien's Law here, which tells us that a hot object's temperature, T, is roughly related to the wavelength of peak emission, $\lambda_{\rm max}$, by

$$\frac{\lambda_{\rm max}}{1 \mu m} = \frac{3000 \text{ K}}{T}. \tag{6}$$

Star (a) has $T_a \approx T_\odot \approx 6000$ K, star (b) has $T_b \approx 3100 \text{ K} \approx T_\odot/2$, and star (c) has $T_c \approx 10 T_\odot \approx 60,000$ K. Since the Sun's radiation peaks at roughly $\lambda_{{\rm max},\odot} \approx (3000/6000) \approx 0.5\,\mu$m = 500 nm, we expect these stars to peak at roughly: (a) 500 nm, (b) twice that, or 1000 nm = 1 μm, and (c) a tenth that, or 50 nm.

(e) Estimate the energy of a single photon at each of those wavelengths, above. [4 pts]

Solution: We just need to recall that the energy of a single photon with frequency ν ("nu") is $E = h\nu$, where h is Planck's constant, and that $nu = c/\lambda$, and then calculate E accordingly.

Alternatively, one can use the handy trick that the energy of a photon measured in electron-Volts (eV) is roughly

$$\frac{E}{1\ \mathrm{eV}} \approx \frac{1\ \mu\mathrm{m}}{\lambda}. \tag{7}$$

From the wavelengths above, this would imply photon energies of $\boxed{2\ eV, 1\ eV, and 20\ eV}$. With 1 eV $\approx 1.6 \times 10^{-19}$ J, this corresponds to photon energies of roughly $\boxed{(3.2, 1.6, 32) \times 10^{-19}\ \mathrm{J}}$.