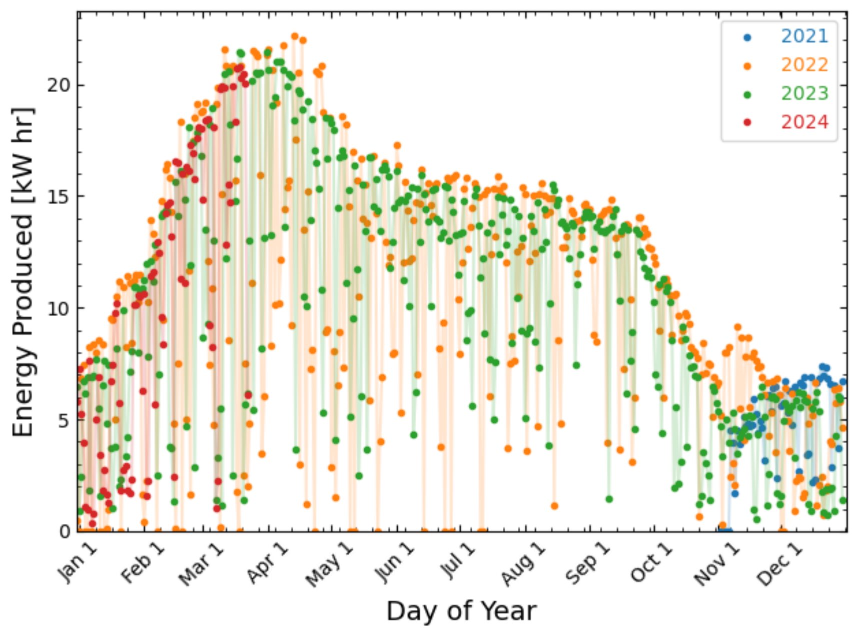
UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2024

Problem Set 7
Due: Friday, April 5, 2024, 10am Kansas Time
This problem set is worth **42 points**.

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

1. **Solar Energy [16 pts]**

   Figure 1 below shows the daily energy produced by Prof. Crossfield's rooftop solar panels over the past few years.



   (a) Describe the general trends you observe in this plot. [4 pts]

   Solution: Mention at least 3 relevant things. Examples include:

   i. Energy production is roughly constant from one year to the next.
   ii. Energy production follows a rough overall trend (sunny days) with a smattering of lower-energy (cloudy) days.
   iii. Energy production peaks in the springtime. [It drops after that because leaves grow out on all the trees!]
   iv. Energy production peaks in lowest in October/November. [With a slight rebound after the leaves all fall.]

   (b) Calculate the Solar Constant, the typical flux of sunlight incident on the Earth, in W/m^2. [3 pts]

   Solution: We recall (or look up!) that the Solar Constant is just the flux of sunlight incident on the Earth. Since the Sun's luminosity is emitted in all directions, all that energy spreads out along the surface of a sphere of radius $r = 1$ AU, and the total flux is then:

$$F = \frac{L}{4\pi r^2} \approx \frac{4 \times 10^{26}\ \mathrm{W}}{12 \times (1.5 \times 10^{11}\ \mathrm{m})^2} \approx \frac{10^{26}\ \mathrm{W}}{6 \times 10^{22}\ \mathrm{m}^2} \approx \boxed{1500\ \mathrm{W/m}^2}. \quad (1)$$

(c) Prof. Crossfield's solar panel system has a total collecting area of roughly 20 m^2. Estimate the maximum power (in W) that you might expect the panels to produce. [2 pts]

Solution: Since power $P = F \times A$, with F as the Solar Constant and A the collecting area we might naively expect a peak power of

$$P_{\max} \approx \left(1500 \text{ W m}^2\right) \times \left(20 \text{ m}^2\right) \approx \boxed{30,000 \text{ W} = 30 \text{ kW}}. \tag{2}$$

(d) Using your estimate of the maximum power, estimate the total energy (in kW-hr, killowatt-hours) that might be produced in a day. [3 pts]

Solution: Energy is power times time. But the Sun is only up for half of the day, and it isn't always shining directly down on the solar panels. So let's assume a factor-of-two penalty for the Sun's angle: 6 hours of sunlight per day. Then we have

$$E_{\text{day,max}} \approx (30 \text{ kW})(6 \text{ hr}) \approx \boxed{180 \text{ kW}}. \tag{3}$$

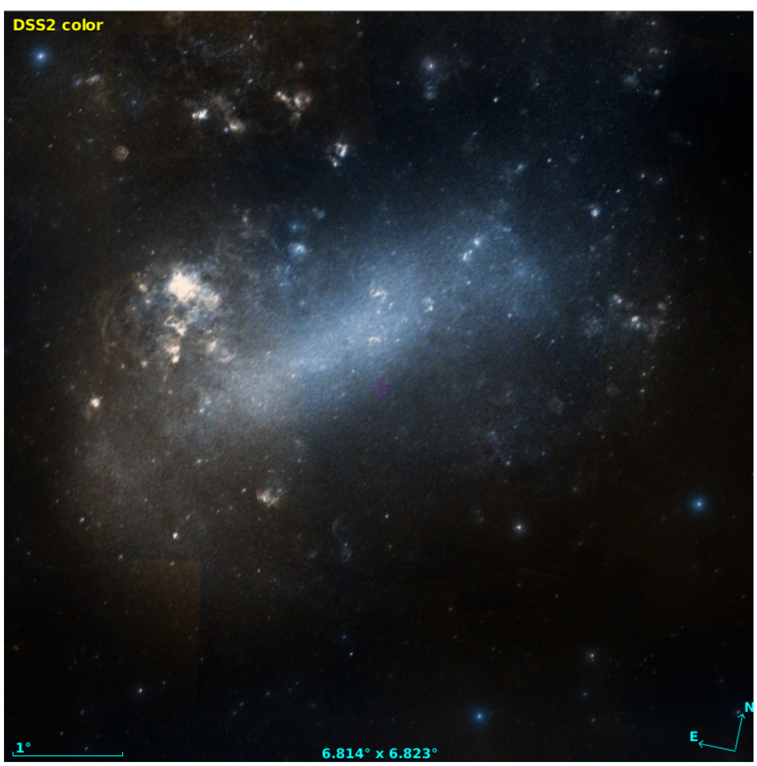
(e) In fact, the system never produces more than about 5 kW of power at peak, and rarely more than $\sim$20 kw-hr of energy per day, at maximum. Describe why these numbers are significantly lower than your rough estimates. [4 pts]

Solution:

Prof. Crossfield still wonders about this, himself! Some factors are likely (i) sun angle (the roof points due South, tilted up at a roughly 45^o angle); (ii) additional losses due to shade from big neighboring trees; (iii) limited total efficiency of the solar panels, spec'ed at $\sim$20%.

2. **A Galaxy (not so) Far, Far Away [26 pts]**

The Large Magellanic Cloud (LMC) is a dwarf/irregular galaxy fairly near to the Milky Way. Here is its selfie, in Figure 2:



(a) The total apparent brightness of the LMC at visible wavelengths is a visual magnitude of ~0.1, corresponding to roughly 10^8 photons/sec/m^2/nm. Assume that all these photons are visible-wavelength, and then estimate the observed flux density, F_λ, from the LMC in W/m^2/nm. [5 pts]

Solution:

[3 pts] We need to convert from photons/s to W (J/s), so we need the energy of a photon. Assuming visible wavelengths, $\lambda \sim 500$ nm and the energy of a visible-light photon is

$$E_\gamma = \frac{hc}{\lambda} \approx \frac{\left(2/3 \times 10^{-33} \text{ J s}\right)\left(3 \times 10^8 \text{ m/s}\right)}{500 \times 10^{-9} \text{ m}} \approx \frac{2 \times 10^{-25} \text{ J m}}{5 \times 10^{-7} \text{ m}} \approx 0.4 \times 10^{-18} \text{ J}. \quad (4)$$

[2 pts] Thus the flux density is

$$F_\lambda = \left(10^8 \text{ photons/sec/nm/m}^2\right)\left(4 \times 10^{-19} \text{ J/photon}\right) \approx \boxed{4 \times 10^{-11} \text{ W/nm/m}^2}. \quad (5)$$

(b) Assume further that all energy from the LMC is radiated at visible wavelengths, and then estimate the total flux observed from the LMC, in W/m^2. [4 pts]

Solution: Since $F \approx F_\lambda \times (\Delta\lambda)$, we need to estimate a wavelength interval $\Delta\lambda$.

[2 pts] Visible light spans roughly 400–700 nm, so assume $\Delta\lambda \approx 300$ nm.

[2 pts] Then we simply have

$$F \approx \left(4 \times 10^{-11} \text{ W/nm/m}^2\right)(300 \text{ nm}) \approx \boxed{1.2 \times 10^{-8} \text{ W/m}^2}. \quad (6)$$

(c) The parallax to the LMC is roughly 20 μas. Estimate the distance to the LMC, in pc. [2 pts]

Solution: By now we're familiar enough with this to recall that:

$$d = \frac{1''}{\theta_{\text{parallax}}} \text{ pc} = \frac{1''}{(20 \times 10^{-6})''} \text{ pc} = \boxed{50,000 \text{ pc}}. \quad (7)$$

(d) Use your value for the distance to estimate the total luminosity of the LMC, in both W and $L_\odot$. [4 pts]

Solution:

[2 pts] The LMC (like all good astronomical objects) must be radiating roughly equally in all directions. So at its distance of $d = 50$ kpc, its luminosity is spread over the area of a sphere with that same radius. Thus $L = F \times \left(4\pi d^2\right)$.

[2 pt] Since 50 kpc $\approx 1.5 \times 10^{21}$ m, we thus have

$$L \approx \left(1.2 \times 10^{-8} \text{ W/m}^2\right)\left(12 \times \left(1.5 \times 10^{21} \text{ m}\right)^2\right) \approx \boxed{3 \times 10^{35} \text{ W} \approx 10^9 L_\odot}. \quad (8)$$

(e) Given the image of the LMC, estimate its angular diameter (in deg), physical diamer (in pc), and angular area (solid angle, in $\square^o$). [6 pts]

Solution:

[2 pts] Very roughly, from the image the LMC's angular diameter seems to range from $\boxed{\sim 2^o}$ (short axis) to $\boxed{\sim 8^o}$ (long axis).

[2 pts] Taking an average angular diameter of $\theta \approx 5^o \sim 0.1\ rad$, with the LMC at $d \approx 50$ kpc away we have a physical diameter D of

$$D = (50,000 \text{ pc})0.1 \approx \boxed{5,000 \text{ pc}}. \quad (9)$$

This number is a bit smaller than that given on Wikipedia, but their number counts even the very faint, wispy outskirts of the LMC. And anyway, ours is just a rough estimate.

[2 pts] For the solid angle subtended by the LMC and using the dimensions estimated above: if we assume a rectangle we get 16 $\square^o$ and if we assume an ellipse we get $\sim 12\ \square^o$. So roughly, $\boxed{\Omega \approx 12 - 16\ \square^o}$.

(f) Use your estimate of the LMC's solid angle to estimate its average surface brightness, I_λ, in W/m^2/nm/$\square^o$. How does this compare to the value of $\sim 10^{-13}$ W/m^2/nm/$\square^o$ that we estimated for M31 (the Andromeda Galaxy) in class? [5 pts]

Solution:

[2 pts] We recall that $I_\lambda \approx F_\lambda/\Omega$, so with $\Omega \approx 14\ \square^o$ we have

$$I_\lambda \approx \frac{4 \times 10^{-11}\ \mathrm{W/nm/m^2}}{14\ \square^o} \approx \boxed{3 \times 10^{-12}\ \mathrm{W/nm/m^2/\square^o}}. \tag{10}$$

[3 pts] This number is pleasantly close to the value we found for Andromeda (within a factor of three). Thus even though Andromeda is $\sim$20$\times$ further away, much larger, and much more luminous the two galaxies have roughly comparable surface brightness. This shouldn't surprise us so very much, since after all I_λ is independent of distance. So we would have gotten the same value, even if the LMC were 100$\times$ further away! (If it were 100$\times$ closer, that would be bad news for the Milky Way!).