

**Problem Set 6**

**Due:** Wednesday, October 22, 2025, in class  
This problem set is worth **23 points** (plus 7 bonus points).

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

**Journey to the Center of a Star**

For this problem, you have two options: use a computer to perform numerical integration (via programming language or, as shown in lecture, a spreadsheet), or alternatively use mathematical integration (calculus). If you use numerical integration, be sure to use  $>20$  layers so your results will be reasonably accurate. Regardless: remembering when making plots for the question below that any good plot has labeled axes!

1. [5 pts] In lecture we discussed at some length how we can model the physical conditions in a star's interior. Assume the star has a slightly more realistic density profile (valid from  $0 \leq r \leq R_*$ ) of

$$\rho(r) = \rho_c \left( (r/R_*)^2 - 2r/R_* + 1 \right) = \rho_c \left( \frac{r}{R_*} - 1 \right)^2 \quad (1)$$

Plot  $\rho(r)$  over the full range from  $r = 0$  to  $r = 2R_*$ . Discuss why this density profile might be slightly more realistic than the constant-density model we assumed in class.

2. [5 pts] Show that the Enclosed Mass profile  $M_{enc}(r)$  within this star is equivalent to the expression

$$4\pi\rho_c r^3 \left( \frac{r^2}{5R_*^2} - \frac{r}{2R_*} + \frac{1}{3} \right). \quad (2)$$

Then plot  $M_{enc}(r)$ .

3. [5 pts] Calculate the gravitational acceleration profile  $g(r)$  inside the star, using the expression above for  $M_{enc}(r)$ . Then plot  $g(r)$ .

4. [3 pts] Starting with the equation of hydrostatic equilibrium ( $dP/dr = -\rho(r)g(r)$ ), we could continue in this vein and calculate the internal pressure and temperature of the star – but things would quickly get really messy.

Instead, we can make a rough approximation to get a sense of the conditions inside the star, by assuming that  $dP/dR \approx P_c/R_*$  (i.e., the pressure at the center of the star divided by the star's radius), and further assuming density and gravity are constant:  $\rho(r) = \rho_{avg}$  and  $g(r) = g_{surface}$ .

Under these simplifying assumptions, the pressure at the center of the star is just  $P_c \approx \rho_{avg} g_{surf} R_*$ . Calculate a numerical value of  $P_c$  (in SI units) for the Sun, and for a red dwarf with  $M_*/M_\odot = R_*/R_\odot = 0.3$ . How do these compare to the atmospheric pressure here on Earth?

5. [5 pts] Assume that our star is an ideal gas made entirely of hydrogen atoms. In this case, derive a symbolic expression for the temperature  $T_c$  at the center of a star in terms of its pressure and mass density.

Then, calculate a numerical value for the central temperature of both the Sun and the M dwarf described above. How do these compare to the surface temperatures of these stars?

6. BONUS [7 pts]: Use the equation of hydrostatic equilibrium to calculate the full pressure profile,  $P(r)$ , throughout the star described in parts 1–3 above. Plot  $P(r)$ .