UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 391) — Prof. Crossfield — Spring 2026

Problem Set 4
Due: Friday, Feb 27, at the start of class
This problem set is worth **40 points**.

As always, be sure to: show your work, circle your final answer, and use the appropriate number of significant figures.

1. **Stellar Lifetimes [5 pts]** For each of the following stars, estimate its main-sequence lifetime. Show your work.

   (a) A star with initial, main-sequence mass of 1 $M_\odot$
   (b) A star with initial, main-sequence mass of 20 $M_\odot$
   (c) A star with initial, main-sequence mass of 2 $M_\odot$
   (d) A star with initial, main-sequence mass of 0.3 $M_\odot$

   Solution: The mass-luminosity relation is $L_* \propto M_*^{3.5}$, which we have usually approximated in A391 as $L_* \propto M_*^4$. From this (and that the total available energe $E \propto M_*$), we find that the main-sequence lifetime is roughly

   $$t \propto \frac{E}{L} \propto \frac{M_*}{M_*^4} \propto M_*^{-3}. \tag{1}$$

   To turn this into a proportionality, we recall that the Sun has a main-sequence lifetime of roughly 10 Gyr. This is because it fuses H to He at 0.7% efficiency, and will fuse roughly 10% of its total available fuel. Thus

   $$t_{MS,\odot} \approx \frac{0.1 \times 0.007 \times M_\odot c^2}{L_\odot} \approx 10 \text{ Gyr}. \tag{2}$$

   So for stars of main-sequence mass 1, 20, 2, and 0.3 $M_\odot$ we would scale that 10 Gyr age by factors of 1^3, 20^{-3}, 2^{-3}, and 0.3^{-3}, respectively.

$M_*/M_\odot$	scale factor	t_{MS}/Gyr
1	1	10
20	1.25×10^{-4}	0.125×10^{-3}
2	1/8	1.25
0.3	37	370

2. **Nuclear binding energies [15 pts]**

   For each of the following nuclear reactions, look up the mass of each of the following nuclei (e.g., on Wikipedia) and calculate ΔM, the change in mass from the ingredients to the products, in atomic mass units (amu). Then, via $\Delta E = \Delta M c^2$ (and/or by noting that $[1\text{amu}\, c^2 \approx 931.494\,\text{MeV}]$): (i) compute the amount of energy released or absorbed by each reaction, (ii) state explicitly whether energy is released or absorbed ($\Delta E > 0$ means energy is released), and (iii) calculate the overall efficiency of each reaction.

   WARNING: don't just read the masses of these nuclei off of the periodic table — the masses listed there are averages over all isotopes of that particular element. Instead, you need to look up the mass of the particular isotope of the particular element listed below. For example: carbon on the periodic table is listed as having a mass of 12.011 amu... but the mass of Carbon-12 (^{12}C) is *exactly* 12.0000..., by definition. These small differences are important when it comes to nuclear reactions!

   (a) $4\,p \rightarrow {}^4_2\text{He}$ (the total p-p chain)
   (b) $3\,{}^4_2\text{He} \rightarrow {}^{12}\text{C}$ (the triple-α reaction)
   (c) ${}^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{24}\text{Mg}$

(d) $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{16}\mathrm{O} + 2{}^{4}\mathrm{He}$

(e) $^{19}\mathrm{F} + {}^{1}\mathrm{H} \rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He}$

(f) $^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + e^{+} + \nu$

(g) $56\, p \rightarrow {}^{56}\mathrm{Fe}$ (the full process, occurring only in the most massive stars)

Solution:

(a) $4\, p \rightarrow {}^{4}_{2}\mathrm{He}$
$\Delta M = 4 \times 1.0073 - 4.0026 = 0.027$ amu
$\Delta E = 25$ MeV
$\epsilon = \Delta m/m \approx 0.027/4 \approx \boxed{0.007}$.

(b) $3\, {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$
$\Delta M = 3 \times 4.0026 - 12 = 0.0078$ amu
$\Delta E = 7.3$ MeV
$\epsilon = \Delta m/m \approx 0.0078/12 \approx \boxed{0.0007}$.

(c) $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}$ ΔE = 13.96 MeV
$\epsilon = \Delta m/m \approx 0.015/24 \approx \boxed{0.0006}$.

(d) $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{16}\mathrm{O} + 2{}^{4}\mathrm{He}$ ΔE = -0.112 MeV; energy is absorbed!
$\epsilon = \Delta m/m \approx -0.0002/24 \approx \boxed{-0.0002}$.

(e) $^{19}\mathrm{F} + {}^{1}\mathrm{H} \rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He}$ ΔE = 8.129 MeV
$\epsilon = \Delta m/m \approx 0.0074/20 \approx \boxed{0.0004}$.

(f) $^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + e^{+} + \nu$ ΔE = 1.445 MeV $\epsilon = \Delta m/m \approx 0.0005/2 \approx \boxed{0.00025}$.

(g) $56p \rightarrow {}^{56}\mathrm{Fe}$
$\Delta M = 56 \times 1.0073 - 55.9349 = 0.474$ amu
$\Delta E = 441$ MeV
$\epsilon = \Delta m/m \approx 0.47/56 \approx \boxed{0.008}$.

All the reactions are exothermic (producing energy) except for (d).

3. **Final Fates of Stars [20 pts].** Starting with its life on the main sequence, enumerate and describe the main stages in the life of a star with an initial (main-sequence) mass of:

(a) $0.2M_{\odot}$
(b) $1M_{\odot}$
(c) $2M_{\odot}$
(d) $10M_{\odot}$
(e) $20M_{\odot}$
(f) $40M_{\odot}$

Solution:

(a) $0.2M_{\odot}$: The star will slowly fuse H to He while on the Main Sequence throughout its entire life (which will be for many tens of Gyr). The star is fully convective and so fully and homogeneously mixed, and will continue fusing H until all is used up. Then it will just slowly fade away.

(b) $1M_\odot$: (i) Stars like the sun will fuse H to He in their core for their MS lifetime. (ii) Once the core H is used up, the star will become a red giant. (iii) The outer layers will be lost, and the core will remain as a hot He white dwarf (surrounded by a planetary nebula). The WD will then slowly cool and fade away.

(c) $2M_\odot$: Steps (i) and (ii) will be similar to the $1M_\odot$ star, but this star will likely subsequently (iii) fuse He to C in its core, perhaps while still fusing H to He in an intermediate shell. (iv) The star will still lose its outer layers and end up as a white dwarf, likely a Carbon white dwarf.

(d) $10M_\odot$: Steps (i), (ii), and (iii) will be similar to the $2M_\odot$ star, but further fusion in the core (and additional shell-burning layers) will result. (iv) Eventually an Fe core will be left which cannot support itself, and (v) the core will collapse and the star will explode in a "core-collapse supernova." (vi) Only a neutron star will be left (surrounded by a hot, thin expanding cloud of debris).

(e) $20M_\odot$: This star will likely follow the same steps as the $10M_\odot$ star above, but its overall lifetime will be shorter.

(f) $40M_\odot$: This star will follow most of the same steps as the $10M_\odot$ star, but (vi) its collapse will be so unstoppable that the central regions will fall inside an event horizon, leaving a Black Hole behind.