**UNIVERSITY OF KANSAS**
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2022

**Problem Set 2**
**Due**: Wednesday, Sep 14, 2022, before the start of class (by 1000), by email.
This problem set is worth **52 points**.

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **How Do You Say "Absorption" in Astrophysics? [12 pts]**
   (a) Define $\alpha_\lambda$, extinction coefficient (in words, not just an equation!), and give its typical units. [3 pts]
   (b) Define $\sigma_\lambda$, cross-section (in words, not just an equation!), and give its typical units. [3 pts]
   (c) Define $\kappa_\lambda$, opacity (in words, not just an equation!), and give its typical units. [3 pts]
   (d) Define $\tau_\lambda$, the optical depth (in words, not just an equation!), and give its typical units. [3 pts]

2. **A Hazy Morning [15 pts]** On a given day the weather report mentions that visibility is about one km. Take that as the distance at which $\tau_\lambda \approx 1$:
   (a) Estimate the atmosphere's extinction coefficient $\alpha_\lambda$ that morning. [3 pts]
   (b) If the visibility is mainly limited by smoke from nearby fires, estimate the effective cross-section $\sigma_\lambda$ of the smoke particles and the number density $n$ of these particles in the atmosphere. [4 pts]
   (c) Given the typical density $\rho$ of air at sea level, estimate the opacity $\kappa_\lambda$ of the smoky air (in SI units). [4 pts]
   (d) Now assume that this same smoke layer is confined to a thin layer near the ground with a thickness of just 100 m (due to some unknown atmospheric phenomenon; the reason isn't important here). What is the optical depth at visible wavelengths when looking straight overhead? and when looking up at a $45^o$ angle (halfway between straight up and parallel to the ground)? [4 pts]

3. **Radiation Quantities [15 pts]** Consider two astronomical objects: an Earth-size planet with $T \approx 300$ K and a Sun-like star $T \approx 5800$ K. Assume that both objects radiate as blackbodies. Recalling that $F_\lambda = \pi I_\lambda \left(\frac{R}{d}\right)^2$ (Rybicki, Eq. 1.13; where $R$ is the object's radius and $d$ is the distance from the object's center):
   (a) Plot the flux density, $F_\lambda$, at the *surface* of each of these objects. [4 pts]
   (b) Plot the flux density, $F_\lambda$, at a distance of 10 parsecs (pc) from each of these objects. [4 pts]
   (c) Plot the wavelength-dependent planet-star flux ratio, $F_{\lambda,p}/F_{\lambda,*}$, of these two blackbody spectra. What does this plot suggest about the best wavelengths to look for radiation emitted by Earth-like planets in other Solar systems? [4 pts]
   (d) Calculate the bolometric (wavelength-integrated) planet-star flux ratio, $F_p/F_*$. [3 pts]

4. **A Very Simple Scattering Model [10 pts]** As we talked about in class, very large objects (like basketballs) absorb visible light based on their geometric cross-section, $\sigma_\lambda = \text{constant}$. But for small objects (or very long wavelengths), when $\lambda \gg r$, we enter the *Rayleigh Scattering* regime where $\sigma_\lambda \propto \lambda^{-4}$.

   Construct and write down an approximate, continuous expression for $\sigma_\lambda$, and then plot it for a range of $\lambda$ extending from $\lambda < r$ to $\lambda > r$.

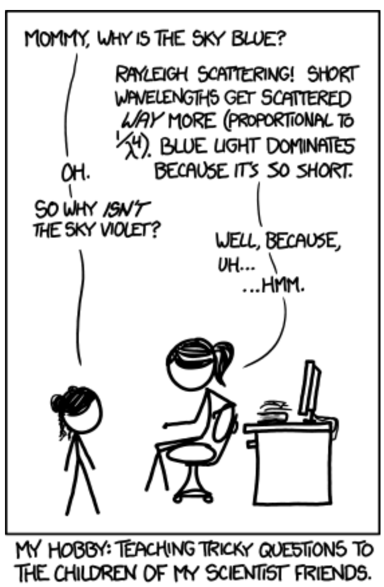


Figure 1: A cruel trick involving Rayleigh scattering, from `https://xkcd.com/1145/`. Bonus points if you answer this child's second question.