UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2022

Problem Set 3
Due: Wednesday, Sep 21, 2022, before the start of class (by 1000), by email.
This problem set is worth **49 points**.

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **Equation of Radiative Transfer [16 pts]**.

   (a) Write the general form of the Equation of Radiative Transfer (not the solution to the EoRT; also, ignore terms of directionality such as μ or θ, which we haven't discussed in lecture yet). Thoroughly explain the meaning of all terms, and the meaning of the overall equation. [5 pts]

   (b) Show that if emission is negligible, that the solution to the equation of radiative transfer is

   $$I(\tau) = I_0 e^{-\tau}. \quad (1)$$

   Explain what this expression means, in words. [3 pts]

   (c) Show that if the source function is constant and absorption is negligible, the solution is instead

   $$I(\tau) = I_0 + \tau S. \quad (2)$$

   Explain what this expression means, in words. [3 pts]

   (d) Show that if the source function is constant and neither absorption nor emission can be ignored, the general solution is

   $$I(\tau) = I_0 e^{-\tau} + S\left(1 - e^{-\tau}\right). \quad (3)$$

   Explain what each term in this expression means, in words. [5 pts]

   (e) In Eq. 3, what is the emergent intensity $I(\tau)$: (i) if $\tau = 0$? (ii) if $\tau \gg 1$? (iii) if $0 < \tau \ll 1$? [5 pts]

2. **Light-Absorbing Clouds, Part I [14 pts]** In this and the next problem, you will consider how a few types of clouds overhead affect the amount of starlight that reaches a ground-based astronomical observatory. In all cases, assume that the cloud does not emit (or reflect, or scatter) any light. It only absorbs.

   First, consider a homogeneous cloud deck: it extends all the way to the ground ($z = 0$) and up to an altitude of $z = H$; above this the cloud is gone and above it the sky is perfectly clear. The cloud has a constant extinction coefficient of α_λ throughout.

   A distant star's light, with intensity $I_{\lambda,0}$ at the top of the atmosphere, is coming down toward your observatory.

   (a) How much optical depth has the starlight passed through when it first reaches the top of the cloud, at $z = H$? What fraction of the incident starlight ($I_{\lambda,0}$) reaches altitude $z = H$? [2 pts]

   (b) How much optical depth has the starlight passed through when it reaches your observatory, at $z = 0$? What fraction of the incident starlight reaches altitude $z = 0$? [4 pts]

   (c) Write a general expression for $\tau_\lambda(z)$, for any altitude $z \leq H$. (Make sure that you get the expected behavior for small or large τ!). [4 pts]

   (d) Write a general expression for $I_\lambda(z)/I_{\lambda,0}$, the fraction of starlight reaching any altitude $z \leq H$. [4 pts]

3. **Ozone [19 pts]**. The Hartley band of the ozone molecule (O_3) is a broad absorption band in the UV, and its cross-section[1] has an approximately Gaussian functional form of

$$\sigma_\lambda = \left(10^{-17}\ \mathrm{cm}^2\right) \exp\left[-\frac{(\lambda - 255\ \mathrm{nm})^2}{2\,(17\ \mathrm{nm})^2}\right]. \tag{4}$$

(a) Plot the ozone cross-section across the full width of this absorption band. (For this and the other plots you will make below, use a wavelength range no narrower than 200–310 nm). [3 pts]

(b) Ozone is mainly found in the Earth's stratosphere, where a (very!) rough average for the number density of ozone particles is $n \sim 3 \times 10^{11}\ \mathrm{cm}^{-3}$. Calculate and plot the absorption coefficient α_λ for ozone across the full width of the Hartley band. [4 pts]

(c) Earth's ozone is mainly concentrated at altitudes from 15–35 km above the Earth's surface. Assume that n is constant with altitude in that range, and then calculate and plot the optical depth τ_λ of the Hartley band in Earth's ozone layer. [4 pts]

(d) Considering only this absorption band (and no other atmospheric absorption features), (i) calculate and plot how much incident Sunlight reaches the surface of the Earth across this ozone absorption band, i.e. as a function of wavelength. What fraction of light reaches the Earth's surface (ii) in the central 'core' of the Hartley band, around 255 nm, and (iii) far in the 'wings' of the band, say at 320 nm? [4 pts]

(e) Fig. 1 shows the transmission of light through Earth's atmosphere across a much wider range of wavelengths. Explain how one might extend the analysis you just completed (i.e., one absorption band of one molecule over a narrow wavelength range) to generate a more complete transmission model such as this. [4 pts]

[1] See e.g., `http://vpl.astro.washington.edu/spectra/o3uvimages.htm`.

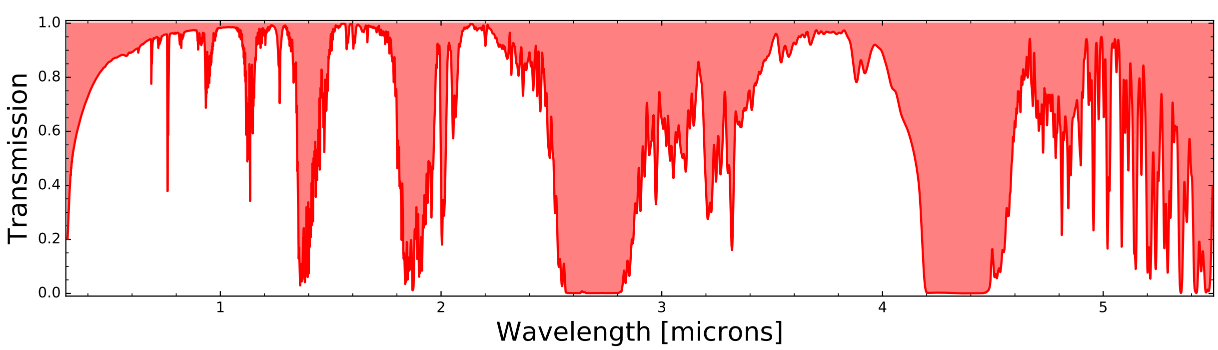

Figure 1: Transmission of the Earth's atmosphere vs. wavelength over optical and near-infrared wavelengths. Shaded wavelengths are more opaque, while white wavelengths are more transparent.