UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2022

Problem Set 4
Due: Friday, September 30, 2022, at the start of class (by 1000).
This problem set is worth **50 points**.

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **The Glowing Infrared Sky [30 pts]**. Assume you are observing from a ground-based infrared observatory, when unusual weather conditions result in an atmospheric temperature T that constant from ground level way up until the air is too thin to matter any more.

   (a) The condition of hydrostatic equilibrium in an atmosphere (which we will learn about later) implies that in such an atmosphere, the atmospheric pressure varies as

   $$P(z) = P_{\mathrm{ground}} e^{-z/H}, \tag{1}$$

   where z is the altitude above the ground, $H \approx 8$ km is the Earth's typical atmospheric scale height (the e-folding scale), and P_{ground} is the pressure at ground level. Show that if the Earth's atmosphere is an ideal gas, that the number density of particles also scales as $n(z) \propto e^{-z/H}$. [3 pts]

   (b) Water vapor is one of the dominant infrared opacity sources. Download from `https://crossfield.ku.edu/files/h2o_crosssection_lores.csv` a file containing the cross-section (σ_λ, in cm^2) of an H_2O molecule as a function of wavelength (in μm). Plot σ_λ (use a logarithmic vertical axis, to show the full dynamic range). [4 pts]

   (c) If a fraction f of all particles in the atmosphere are water vapor molecules, what is the number density $n_{\mathrm{H2O}}(z)$ – i.e., as a function of altitude? Assume that the atmosphere is uniformly mixed throughout. [3 pts]

   (d) If your observatory is at sea level and f=1% and the atmospheric temperature is $T = 274$ K throughout, what is $\langle n_{H2O} \rangle$, the *average* number density of H_2O molecules between 0–8 km? [3 pts]

   (e) Now assume that from $z = 0$ to $z = H \approx 8$ km the atmosphere is uniform and homogeneous: with H_2O number density $\langle n_{H2O} \rangle$, temperature $T = 274$ K, and thickness 8 km (with only empty space above; we're simplifying to keep things tractable). Using the downloaded file of σ_λ, calculate and plot both the absorption coefficient α_λ and the optical depth τ_λ as a function of wavelength. [6 pts]

   (f) If we make the reasonable assumption that the atmosphere is in LTE and our telescope is pointing at an empty patch of space overhead, calculate and plot (vs. wavelength) the intensity I_λ radiated by the warm sky. (This sky emission will consequently be an annoying source of background 'noise' in any infrared astronomical measurements). On the same graph, overplot the intensity $B_\lambda(T)$ that would be emitted by a T=274 K blackbody. (Hint: apply the general solution to the radiative transfer equation!) [5 pts]

   (g) Compare your plot with that shown in Fig. 1 — it has slightly different vertical units, but fundamentally it's still I_λ. What features look similar between your plot and this one? What looks different (ignore the difference in units)? Discuss and explain the similarities and differences. [6 pts]

2. **Atomic Transitions [20 pts]**

   (a) For a neutral H atom, draw an energy diagram for transitions between all energy levels $n \leq 6$. [5 pts]

   (b) For these same hydrogen transitions ($n \leq 6$) plot on a graph or number line, and label, the locations of all possible photon wavelengths. Use a logarithmic wavelength axis. [5 pts]

(c) If any atom loses all but one of its electrons, the spectrum of that ion (e.g. He^{+}, Li^{++}, etc.) will resemble that of a Hydrogen atom. How will the ion's energy levels and spectrum be similar to that of hydrogen? how will they be different? [4 pts]

(d) Consider a spectrum of a star that your colleague recently observed at their telescope. The spectrum shows a spectral absorption line that neither of you are familiar with. Explain how you might go about identifying the physical source of this absorption line. What would be the challenges in this process? How could you obtain a more certain answer or otherwise test your initial conclusion? [6 pts]

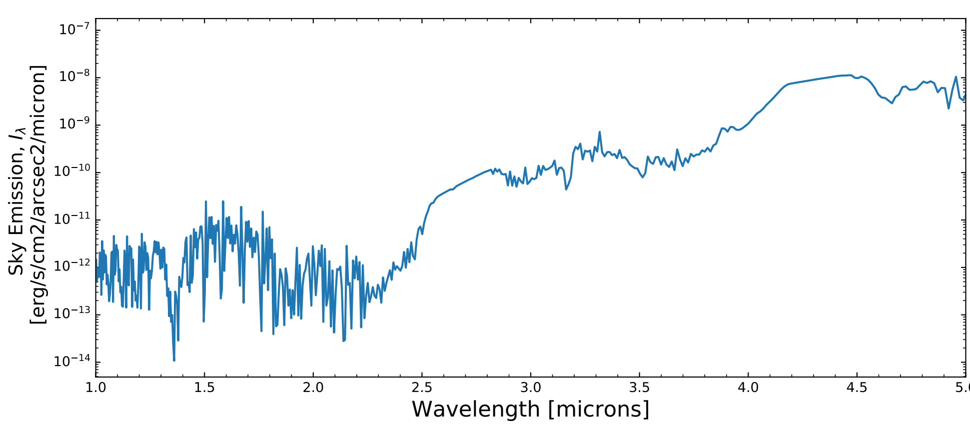

Figure 1: Observed emission spectrum of the glowing infrared night sky over Maunakea Observatory. The vertical units may look a bit unusual, but fundamentally it's still just I_λ.