Chapter Ten

Atmospheric Circulation

10.1 INTRODUCTION

Atmospheric circulation is the large-scale movement of gas in a planetary atmosphere that is responsible for distributing energy absorbed from the star throughout the planetary atmosphere. Curiously, many textbooks on atmospheric radiation omit discussion of atmospheric circulation. Conversely, textbooks on atmospheric circulation often relegate a description of radiation in the atmosphere to one chapter or less. This segregation happens for two reasons. First, for solar system planets, interpretation of spectra in terms of molecular abundances and vertical temperature structure often does not require atmospheric circulation. Second, calculation of atmospheric circulation and dynamics is computationally time consuming, and the timescales of atmospheric dynamics are very different from those of radiation. Atmospheric circulation codes can usually only afford to have a relatively crude radiation scheme.

Atmospheric dynamics focuses on observational and theoretical analysis of motion in the atmosphere on a diverse scale. On Earth, atmospheric circulation models are used both to predict weather and to assess climate change. Atmospheric circulation is also responsible for many large- and small-scale phenomena such as the trade winds, the jet stream, the El Niño and La Niña weather oscillations, hurricanes and tornadoes, and the jet stream. On Jupiter, atmospheric circulation causes high wind speeds in jets and the intricate weather patterns on Jupiter, including the great red spot.

A very fundamental issue is that on all of the solar system planets atmospheric circulation acts to minimize temperature gradients, in part by transporting heat poleward. This means that, despite all of the atmospheric phenomena on all planetary atmospheres created by atmospheric circulation, the longitudinal and latitudinal temperature variation is relatively small, as are the resultant emergent spectra. (One notable exception is a hemispherically integrated spectrum of Earth centered on Earth's cold poles.) Figure 10.1 shows that the latitudinal temperature varies for less than a few degrees for the solar system giant planets. For these planets the 1D temperature profile approach described in (Chapter 9) and earlier chapters is adequate to infer the vertical temperature gradient, surface temperature, and atmospheric composition.

On some planets beyond our solar system, such as hot tidally locked exoplanets, atmospheric dynamics does affect the emergent spectra. We then have a motivation to study atmospheric circulation beyond basic planetary knowledge to use atmospheric dynamics for interpreting spectra. The hot Jupiters are several times closer

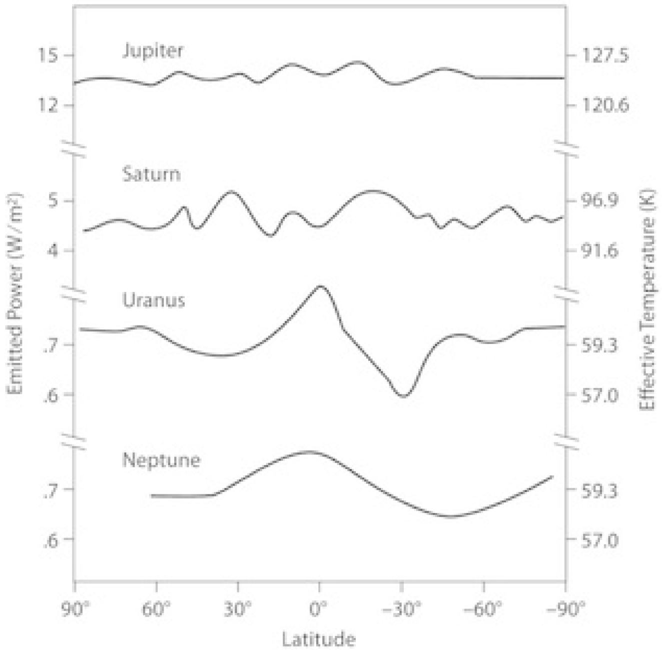

Figure 10.1 Temperature versus latitude on solar system giant planets. The solar system giant planets show almost no temperature change with latitude. Adapted from [1].

to their star than Mercury is to our Sun, receiving 400 times more radiation than Earth and 10,000 times more than Jupiter. The energy from stellar heating dominates any internal flux for hot Jupiters. Together with the permanent day and night sides, this case of stellar forcing has no solar system analog. The fate of the absorbed stellar energy and the global temperature structure of the planet can only be understood via atmospheric circulation. The question of energy redistribution is especially significant for habitable-zone planets orbiting low-mass M stars. Like their hot-Jupiter counterparts, such planets are so close to the star that they are tidally locked, presenting the same face to the star at all times. Whether or not the planet is habitable depends on the role played by atmospheric circulation. By "tidal locking" we are referring to the planet core; an additional complication is to what extent the atmosphere departs from tidal locking. A reasonable picture for planets with thick atmospheres is a tidally locked core with a thick mobile atmosphere.

To complete the picture of the thermal structure and emergent flux characteristics of the tidally locked exoplanet atmospheres, the dynamical response of the atmosphere to heat sources and sinks is the final major process we must consider. These considerations are most significant for tidally locked exoplanets. Due to the complexity and nonlinearity of the atmospheric circulation equations, the full derivation and application of the equations are beyond the scope of this book. Similarly, because of the complexity of the equations, an intuitive, conceptual understanding of atmospheric dynamics is often elusive. In this chapter we will focus mostly on

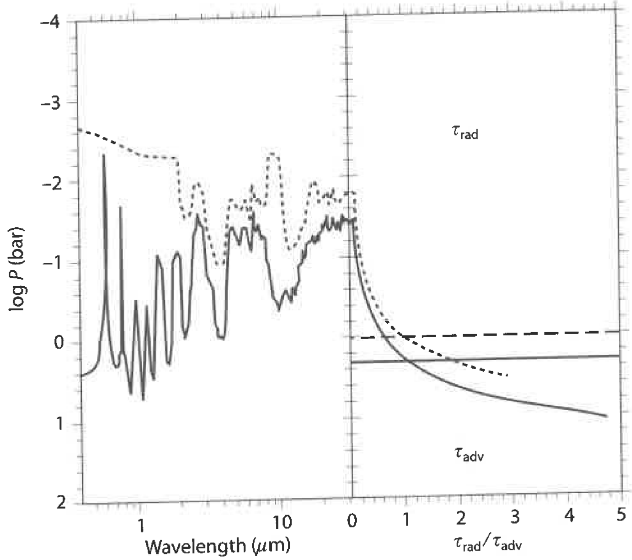

Figure 10.2 Illustration of $\tau_{\rm rad}$ vs. $\tau_{\rm adv}$. Left: pressure (as a proxy for altitude) at optical depth of 2/3 as a function of wavelength. Two different models are shown (cloudy by the dashed curve and clear by the solid curve). Right: altitude dependence of the ratio of the radiative to advective timescales ($\tau_{\rm rad}/\tau_{\rm adv}$). A wind speed U of 1000 m s^{-1} was adopted for illustration; the ratio scales linearly with U so that other values can be considered. Adapted from [3].

timescales and parameters that describe large-scale flow. We will also present a schematic outline of the equations for atmospheric circulation.

10.2 RADIATIVE AND ADVECTIVE TIMESCALES

We begin by estimating the typical timescales that govern whether or not atmospheric circulation is important in causing a longitudinal or latitudinal temperature gradient. This discussion is necessarily oversimplified but helps to illustrate some basic points. Essentially, there is a competition between the radiative and the advective timescales. The radiative timescale ($\tau_{\rm rad}$) is the time for absorbed stellar energy to be reemitted as radiation. The advective timescale ($\tau_{\rm adv}$) is the time for the absorbed stellar energy to be circulated around the planet.

If $\tau_{\rm rad} \ll \tau_{\rm adv}$, the bulk of the absorbed stellar energy will be reemitted before being advected around the planet. In this case, a strong latitudinal and longitudinal temperature gradient are expected to arise. If, in contrast, $\tau_{\rm adv} \ll \tau_{\rm rad}$, the temperature should be much more uniform, because heat would be transported and redistributed efficiently over the entire planet.

The radiative timescale can be estimated by considering an atmosphere layer of thickness Δz that slightly perturbed from radiative equilibrium. Consider an

Table 10.1 Comparison of radiative and advective timescales of some solar system planets.

Planet	$\tau_{\rm rad}$	$\tau_{\rm adv,lon}$	$\Delta T_{\rm lon}$	$\tau_{\rm adv,lat}$	$\Delta T_{\rm lat}$
Venus	years	days	$<$ few K	weeks	few K
Earth	weeks–months	1 day	$\sim$10 K	weeks	20–30 K
Mars	days	1 day	$\sim$50 K	weeks	$\sim$100 K
Jupiter	years–decades	decades	$<$ few K	decades	$<$ few K

Estimated temperature differences in longitude and latitude are also presented. Table adapted from [2] and references therein.

atmospheric layer of pressure thickness ΔP that is out of radiative equilibrium by an amount ΔT. This layer has excess energy per area $\rho c_p \Delta T \Delta z$. Approximating the radiation as black body, the layer will radiate a net excess of $4\sigma_{\rm R} T^3 dT$. Taking the above two statements and using the hydrostatic equilibrium equation gives

$$\boxed{\tau_{\rm rad} \sim \frac{\Delta P}{g} \frac{c_p}{4\sigma_{\rm R} T^3}.} \tag{10.1}$$

Here T is temperature, g is surface gravity, c_p is the specific heat capacity, and $\sigma_{\rm R}$ is the radiation constant. P/g is equivalent to mass/area. This estimate applies only to regions of low optical depth. The advective timescale can be estimated by the planet radius and the characteristic windspeed U,

$$\boxed{\tau_{\rm adv} \sim \frac{R_{\rm p}}{U}.} \tag{10.2}$$

Here U is unknown for exoplanets—indeed the windspeed is a key term one wants to derive from atmospheric circulation models. We now use these timescale estimates to investigate temperature contrasts on planets, at altitudes where the bulk of solar energy is absorbed. We can see from Table 10.1 that, indeed, a longer $\tau_{\rm rad}$ leads to a smaller latitudinal and longitudinal temperature gradient. Although the advective timescales on solar system giant planets such as Jupiter are long, the radiative times are even longer. Furthermore, the fast rotation rate ($\sim$12 hours; see Table A.1) in part means that Jupiter is heated relatively uniformly.

We now turn to an important subtlety in our radiative versus advective timescale analysis. The ratio of the radiative-to-advective timescales depends on the vertical altitude where the stellar radiation is absorbed. This is primarily because $\tau_{\rm rad}$ decreases rapidly deeper into the atmosphere; it depends linearly on pressure, which increases exponentially deeper into the atmosphere (section 9.3.1). In general, the above description of competing timescales is valid for the altitude where the bulk of the stellar energy is absorbed.

Different layers of the atmosphere may have different circulation regimes. Earth, for example, has a relatively small latitudinal and longitudinal temperature difference in the troposphere where we live. High in the thermosphere, however, the day-night temperature difference can be as high as 1000 K. How can we observe different atmospheric layers on exoplanets? Recall that the optical depth is frequency dependent; molecules absorb more strongly at specific frequencies. We explained in previous chapters that this means we can see different layers of the atmosphere depending on the frequency of observation. Figure 10.2 shows the optical depth unity for a hot Jupiter atmosphere to illustrate that optical depth—and hence radiative versus advective regimes—are at different altitudes for different wavelengths.

Can we repeat the radiative versus advective timescale analysis for exoplanets? The main issue in understanding which regime the atmosphere is in is the unknown windspeed U. This value is not known on exoplanets but can either be estimated (in a circular fashion by using $\tau_{\rm rad}$ vs. $\tau_{\rm adv}$ and an estimated temperature) or obtained from computer simulations.

10.3 LARGE-SCALE FLOW AND PATTERNS

Characteristic length scales can tell us something about the big picture of heat transport and weather patterns on a given exoplanet. Before describing two major characteristic scales, we discuss the transformation from an inertial to a rotating reference frame.

10.3.1 The Rotating Reference Frame

In planetary atmospheric circulation it is natural to use the rotating frame of reference, where the so-called inertial or fictitious forces appear. These are the centrifugal and Coriolis forces. Here we will present an outline of the derivation of a transformation from the inertial to the rotating reference frame, that essentially results in the addition of the centrifugal and Coriolos "forces." We will follow [4, 5] in our derivation. Let us take $\mathbf{A}$ as an arbitrary vector in a Cartesian inertial reference frame. The vector is described by

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_j\hat{\mathbf{j}} + A_k\hat{\mathbf{k}}, \tag{10.3}$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are unit vectors. We will describe the same vector in a reference frame rotating with angular velocity $\mathbf{\Omega}$.

$$\mathbf{A} = A'_x\hat{\mathbf{i}}' + A'_j\hat{\mathbf{j}}' + A'_k\hat{\mathbf{k}}'. \tag{10.4}$$

The total Lagrangian derivative of $\mathbf{A}$ in the inertial frame (subscript I) is

$$\begin{aligned}\frac{D_I\mathbf{A}}{Dt} &= \frac{DA_x}{Dt}\hat{\mathbf{i}} + \frac{DA_y}{Dt}\hat{\mathbf{j}} + \frac{DA_k}{Dt}\hat{\mathbf{k}} \\ &= \frac{DA'_x}{Dt}\hat{\mathbf{i}}' + \frac{DA'_y}{Dt}\hat{\mathbf{j}}' + \frac{DA'_k}{Dt}\hat{\mathbf{k}}' + \frac{D_I\hat{\mathbf{i}}'}{Dt}A'_x + \frac{D_I\hat{\mathbf{j}}'}{Dt}A'_y + \frac{D_I\hat{\mathbf{k}}'}{Dt}A'_z.\end{aligned} \tag{10.5}$$

We can use the definition of the derivative in the rotating frame

$$\frac{D\mathbf{A}}{Dt} = \frac{DA'_x}{Dt}\hat{\mathbf{i}}' + \frac{DA'_y}{Dt}\hat{\mathbf{j}}' + \frac{DA'_k}{Dt}\hat{\mathbf{k}}' \tag{10.6}$$

and the cross product terms for the velocity of the unit vectors caused by rotation, e.g.,

$$\frac{D_I\hat{\mathbf{i}}'}{Dt} = \mathbf{\Omega}\times\hat{\mathbf{i}}', \tag{10.7}$$

to write equation [10.5] as

$$\boxed{\frac{D_I\mathbf{A}}{Dt} = \frac{D\mathbf{A}}{Dt} + \boldsymbol{\Omega} \times \mathbf{A}.} \quad (10.8)$$

We can similarly derive a transformation for the rate of change of velocity $\mathbf{U}$ in the rotating frame of reference (again following [4]). We want to find an expression for $\frac{D_I\mathbf{U}_I}{Dt}$. We may use a position vector $\mathbf{r}$ to write

$$\frac{D_I\mathbf{r}}{Dt} = \mathbf{U}_I. \quad (10.9)$$

Applying our transformation equation [10.8] we have

$$\frac{D_I\mathbf{r}}{Dt} = \frac{D\mathbf{r}}{Dt} + \boldsymbol{\Omega} \times \mathbf{r} \quad (10.10)$$

or

$$\mathbf{U}_I = \mathbf{U} + \boldsymbol{\Omega} \times \mathbf{r}. \quad (10.11)$$

We may now again apply our transformation equation [10.8] to $\mathbf{U}_I$ to find

$$\frac{D_I\mathbf{U}_I}{Dt} = \frac{D\mathbf{U}_I}{Dt} + \boldsymbol{\Omega} \times \mathbf{U}_I. \quad (10.12)$$

This equation can be worked out to

$$\boxed{\frac{D_I\mathbf{U}_I}{Dt} = \frac{D\mathbf{U}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{U} - \Omega^2\mathbf{R},} \quad (10.13)$$

where the details are left as an exercise. Here $\boldsymbol{\Omega}$ is taken to be a constant and $\mathbf{R}$ is a vector perpendicular to the axis of rotation. The magnitude of $\mathbf{R}$ is equal to the distance to the axis of rotation.

The second term on the right-hand side of equation [10.13] is known as the Coriolis force or the Coriolis acceleration and the second term is the centrifugal force. Overall, equation [10.13] tells us that the acceleration following the motion in an inertial frame is also described in the rotating frame as the rate of change of relative velocity; and the Coriolis acceleration due to the relative motion; and the centripetal acceleration caused by the rotation of the coordinates [4].

10.3.2 The Rossby Number: Rotation

The Rossby number can tell us whether or not planetary rotation is important for a given phenomenon. Let us take a motion with a length scale L and consider the speed U. We would say the rotation of the planet is important if

$$\frac{L}{U} \geq \frac{1}{\Omega} \quad (10.14)$$

where Ω is the angular velocity of planetary rotation. Rotation is more important at high latitudes than at latitudes near the equator. We therefore replace Ω with the term

$$f = 2\Omega \sin\theta, \quad (10.15)$$

where θ is latitude and f is the Coriolis frequency. The Rossby number is

$$\boxed{R_0 = \frac{U}{Lf}.} \tag{10.16}$$

The Rossby number indicates the importance of rotation on the flow. A small Rossby number (taken to be < 1) means that the effects of planetary rotation are large compared to the fluid motions, and a large Rossby number means the opposite. Let us take the length scale of a planet as R_p and consider a Jupiter-size planet with $R_\mathrm{p} = 7.14 \times 10^7$ m. Jupiter's rotation rate gives a small Rossby number. A hot Jupiter synchronously rotating gives a large number.

The Rossby number depends on L and U. What appears to be small scale from an exoplanet view, such as a 100 m region in an ocean, may still have a small Rossby number. For exoplanets, we are interested in much larger scales only.

10.3.3 The Burger Number: Vertical Stratification

The Burger number is a dimensionless number that indicates the vertical stratification of a fluid, in this case the atmosphere. The Burger number is defined as [e.g., 5]

$$\boxed{\mathrm{Bu} \equiv \left(\frac{L_D}{L}\right)^2,} \tag{10.17}$$

where

$$L_D \equiv \frac{\sqrt{gH}}{|f|}, \tag{10.18}$$

and U, L, and H are characteristic velocity, length, and layer thickness scales, respectively. L_D is the Rossby deformation radius. The Rossby deformation radius is the horizontal length scale at which pressure perturbations are resisted by the Coriolis force.

A Burger number of zero indicates a flow dominated by rotation, whereas a Burger number near 2 indicates a flow dominated by stratification.

10.3.4 The Rhines Scale: Number of Bands

The Rhines scale [5] is the scale at which planetary rotation causes east-west elongation (jets),

$$L_\beta = \pi\sqrt{\left(\frac{2\boldsymbol{U}}{\beta}\right)} \tag{10.19}$$

where U is the eddy windspeed and β is the latitudinal gradient of f,

$$\beta = 2\boldsymbol{\Omega}\frac{\cos\phi}{R_\mathrm{p}}. \tag{10.20}$$

The number of bands, N, on a planet might be approximately described by [6]

$$N \sim \frac{\pi R_\mathrm{p}}{L_\beta}. \tag{10.21}$$

Fluid motions are free to grow in the longitudinal direction, but are confined in the latitudinal direction by the characteristic Rhines scale—in other words, by the gradient of the Coriolis force. The bands confine clouds, and may affect the cloud structures and patterns we can see in a hemispherically integrated exoplanet spectrum.

10.4 ATMOSPHERIC DYNAMICS EQUATIONS

The fundamental equations that govern atmospheric motion come from the conservation laws for momentum, mass, and energy—laws of basic physics. Typically, the assumption of hydrostatic equilibrium (Section 9.3.1) is used to remove the vertical dimension of the momentum conservation equation. An equation of state, taken as the ideal gas law, is also needed. Many textbooks are devoted to the derivation and application of the atmospheric fluid dynamic equations. We aim here to present an outline of derivation of the equations used for exoplanet atmospheric circulation and some of the approximations that lead to models commonly used in exoplanet studies. For our summary outline, we closely follow the introduction to the meterological equations in [4, 7].

We consider an infinitesimal control volume fixed in an Eulerian reference frame in the atmosphere. This is a parallelepiped with a fixed position relative to the moving atmosphere. In this infinitesimal control volume, we will account for conservation of mass, momentum, and energy.

The conservation laws that lead to the atmospheric equations involve the rates of change (i.e., derivatives) of the momenum, density, and energy. Because the atmosphere is moving, the conservation laws must deal with a moving fluid. We are using a fixed volume, and therefore our first step is to relate the local derivative at a fixed point to the "total" derivative to the total rate of change of a variable of interest. We leave it as an exercise to show that

$$\frac{D\mathbf{A}}{Dt} = \frac{\partial \mathbf{A}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{A}, \tag{10.22}$$

where $\mathbf{A}$ is a vector function of interest ($\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are unit vectors). This equation tells us that the total rate of change of $\mathbf{A}$ as the particles move through some velocity field $\mathbf{U}$ is the sum of the local rate of change and the rate of change of $\mathbf{A}$ following the motion. The velocity field $\mathbf{U}$ is

$$\mathbf{U} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}, \tag{10.23}$$

where u, v, and w are conventional notation in atmospheric fluid dynamics, and the unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are conventionally taken to be directed eastward, northward, and upward, respectively.

Remember that we are considering an infinetesimal control volume fixed relative to the moving atmosphere. The derivative expression of interest, therefore (from equation [10.22]) is

$$\boxed{\frac{\partial \mathbf{A}}{\partial t} = \frac{D\mathbf{A}}{Dt} - \mathbf{U} \cdot \nabla \mathbf{A}.} \tag{10.24}$$

10.4.1 Conservation of Momentum

The conservation of momentum begins with the familiar Newton's First Law, in an interial reference frame,

$$\mathbf{F} \equiv \mathbf{f}/m = a = \frac{D_I \mathbf{U}_I}{Dt}, \tag{10.25}$$

where f is the net force, m is the mass, $\mathbf{F}$ is the net force per unit mass (not to be confused with the radiative or convective flux described in previous chapters), $\mathbf{U}$ is the velocity field, and t is time as before. First, recall the expression for $D_I\mathbf{U}_I/Dt$ given in equation [10.13].

Second, we will describe the forces $\mathbf{F}$ on the atmosphere. We have previously discussed the forces in a 1D atmosphere, namely, the forces in the vertical direction as a balance of the pressure gradient and gravity (Section 9.3.1). In 3D the pressure gradient force is a force acting in three dimensions,

$$\nabla P = \frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}}. \tag{10.26}$$

As an acceleration, the pressure force is written $\nabla P/\rho$. In atmospheric fluid dynamics, the gravitational force is written as an effective gravity term that includes the centripetal acceleration term

$$\mathbf{g}_{\text{eff}} = -g\hat{\mathbf{k}} - \Omega^2 \mathbf{R}. \tag{10.27}$$

The remaining forces in the atmosphere are lumped together as frictional forces per unit mass, which we will denote as $\mathbf{F}_{\text{f}}$.

For the conservation of momentum, we have the equation

$$\boxed{\frac{D\mathbf{U}}{Dt} = 2\boldsymbol{\Omega}\times\mathbf{U} - \frac{1}{\rho}\nabla P + \mathbf{g} + \mathbf{F}_{\text{f}}.} \tag{10.28}$$

This is the three-dimensional equation for the conservation of momentum.

We are not completely finished working with the conservation of momentum equations. For further analysis, we further expand the equations into their scalar form using a spherical coordinate system. The complication is in relating the Cartesian coordinate system (x, y, z) in our infinitesimal control volume to the spherical coordinate system (λ, ϕ, z) fixed to the Earth's reference frame, where ϕ is the latitude, λ is the longitude, and z is the vertical distance above the surface of Earth. The complication lies in that the (x, y, z) coordinate system is a function of location and therefore the unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ are also changing with location on Earth's surface. To continue we will adopt relationships between the coordinate systems, and between the unit vectors and latitude, longitude, and vertical direction above the surface, and take a instead of r in a thin shell approximation,

$$\begin{aligned} u &\equiv a\cos\phi\frac{D\lambda}{Dt}, \\ v &\equiv a\frac{D\phi}{Dt}, \\ w &\equiv \frac{Dz}{Dt}, \end{aligned} \tag{10.29}$$

and

$$\frac{D\mathbf{U}}{Dt} = \left(\frac{Du}{Dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a}\right)\hat{\mathbf{i}} + \left(\frac{Dv}{Dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a}\right)\hat{\mathbf{j}} + \left(\frac{Dw}{Dt} - \frac{u^2+v^2}{a}\right)\hat{\mathbf{k}}. \tag{10.30}$$

The interested reader can derive these by consulting the Figures in [4], and other standard atmospheric dynamics textbooks.

We may next expand the Coriolis force by using the definition of the vector cross product and considering $\mathbf{\Omega}$ in terms of the unit vectors: $\mathbf{\Omega} = 0\hat{\mathbf{i}} + 2\Omega\cos\phi\hat{\mathbf{j}} + 2\Omega\sin\phi\hat{\mathbf{k}}$; in other words, the angular velocity has no component parallel to the unit vector $\hat{\mathbf{i}}$:

$$-2\mathbf{\Omega}\times\mathbf{U} = (2\Omega w\cos\phi - 2\Omega v\sin\phi)\,\hat{\mathbf{i}} - 2\Omega u\sin\phi\hat{\mathbf{j}} + 2\Omega u\cos\phi\hat{\mathbf{k}}. \tag{10.31}$$

We finally write an expression for $\mathbf{F}_\mathrm{f}$ as

$$\mathbf{F}_\mathrm{f} = F_{\mathrm{f}x}\hat{\mathbf{i}} + F_{\mathrm{f}y}\hat{\mathbf{j}} + F_{\mathrm{f}z}\hat{\mathbf{k}}. \tag{10.32}$$

We now take the above three equations, as well as our vector notation expressions for ∇P (equation [10.26]) and $\mathbf{g}_\mathrm{eff}$ (equation [10.27]). We substitute these equations into the conservation of momentum equation [10.28] and equate terms in each of the the unit vector directions to find the eastward component of the momentum equations,

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + F_{\mathrm{f}x}, \tag{10.33}$$

the northward component of the momentum equations,

$$\frac{Dv}{Dt} + \frac{u^2\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial P}{\partial y} - 2\Omega u\sin\phi + F_{\mathrm{f}y}, \tag{10.34}$$

and the vertical component of the momentum equations,

$$\frac{Dw}{Dt} - \frac{u^2+v^2}{a} = -\frac{1}{\rho}\frac{\partial P}{\partial z} - \mathrm{g}_\mathrm{eff} + 2\Omega u\cos\phi + F_{\mathrm{f}z}. \tag{10.35}$$

10.4.2 The Conservation of Mass

The conservation of mass is often referred to as the continuity equation. We consider an infinitesimal volume and ask: what is the rate of increase of mass per unit volume $\partial\rho/\partial t$ of the mass inside the volume (i.e., the density change), due to mass flowing in and out of the volume? We start with mass flowing into the volume along the x direction through an area $\delta y\delta z$ (a face of a cube with sides δx, δy, δz). The mass flow rate (in units of kg s^{-1}) into the volume along the x direction through the area $\delta y\delta z$ is

$$\left(\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right)\delta y\delta z, \tag{10.36}$$

and the mass flow rate (in units of kg s^{-1}) out of the volume along the x direction through the area $\delta y \delta z$ is

$$\left(\rho u + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z. \tag{10.37}$$

Subtracting the above two equations, we find the net rate of mass flow through the x-z sides of the volume to be

$$-\frac{\partial(\rho u)}{\partial x}\delta x \delta y \delta z. \tag{10.38}$$

Taking similar expressions for the mass flow along the y and z directions, we have for the net rate of mass flow into and out of the volume

$$\frac{\partial \rho}{\partial t}\delta x \delta y \delta z = -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho u)}{\partial y} + \frac{\partial(\rho u)}{\partial z}\right]\delta x \delta y \delta z. \tag{10.39}$$

We can rewrite the above equation as

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0.} \tag{10.40}$$

This is called the mass divergence form of the continuity equation.

10.4.3 The Conservation of Energy

The law of conservation of energy is used to relate the temperature of the atmosphere to heat sources and sinks. For example, with atmospheric dynamics in a differential layer with thickness Δz there is a net radiative flux

$$\Delta F(z) = F(z) - F(z + \Delta z), \tag{10.41}$$

where the absorbed radiation heats the layer. We may use similar arguments to those preceding the estimate of the radiative timescale (equation [10.1]) to express the layer heating by a rate of temperature change

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_p}\frac{\Delta F(z)}{\Delta z}. \tag{10.42}$$

See, e.g., [9] for more details.

10.4.4 The Ideal Gas Law

The ideal gas law remains the same as before and is included here for completeness.

$$P = \rho R_s T, \tag{10.43}$$

where R_s is the specific gas constant.

10.4.5 Models and the System of Conservation Equations

Atmospheric circulation models are based on the above six fundamental conservation equations: the conservation of mass, conservation of momentum (one equation for each dimension), conservation of energy, and the ideal gas law as the equation

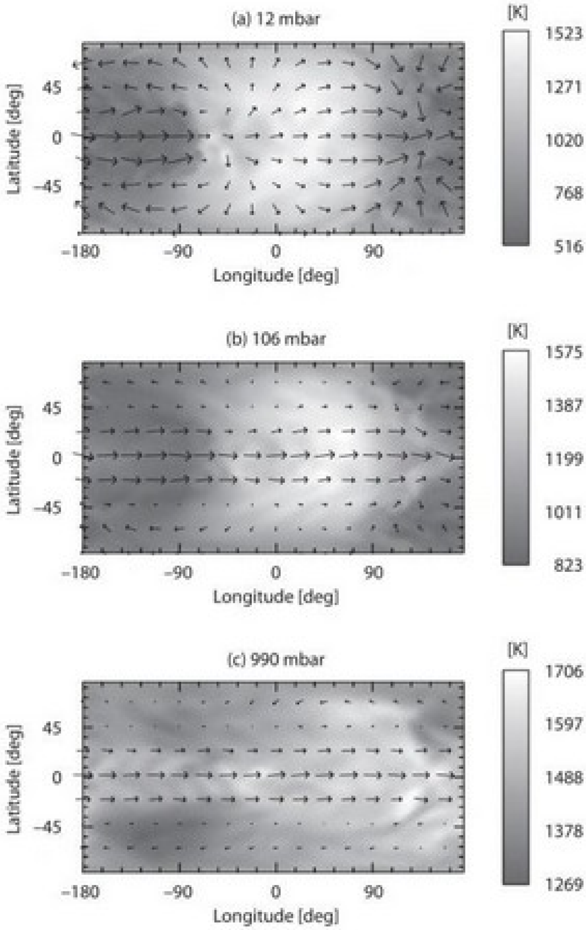

Figure 10.3 Simulated temperature of the tidally locked hot Jupiter HD 209458b. The arrows represent winds. From top to bottom are three different isobars: 1.5, 220, and 19.6 mbar. The substellar point is at (0,0) in longitude and latitude. Adapted from [8].

of state. The system of equations is not closed because of the unknown terms for friction and for the heating rate.

Many variations of conservation equations are used for atmospheric circulation models. Some researchers use the full set of conservation equations. Others take the traditional planetary atmospheres approach resulting from decades of study: the primitive equations. The primitive equations replace the vertical momentum equation with local hydrostatic balance, thereby dropping the vertical acceleration, advection, Coriolis, and metric terms that are generally expected to be less important for the global-scale circulation, such that energy is still conserved. For an example of the primitive equations in a general circulation model as applied to a hot Jupiter exoplanet see Figure 10.3. The term "primitive" refers to the full set of basic equations, before simplification by a suite of approximations. A different modeling

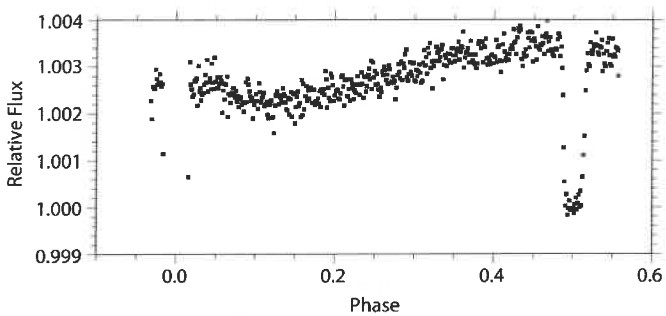

Figure 10.4 Thermal phase curve of HD 189733b. This is a zoomed-in section of Figure 1.4. Adapted from [10].

approach is to use a 2D version of the conservation equations, the shallow water equations. Because of the short timescales involved for radiation transport, atmospheric circulation models traditionally use somewhat elementary radiative transfer schemes.

The primitive equations or even more approximate forms are not easy to implement. Many institutions have developed their own General Circulation Models for public use. Many textbooks spend several chapters continuing the derivation of the conservation equations into different vertical coordinate systems and making many further approximations to make the equations more usable [e.g., 4, 7].

10.5 CONNECTION TO OBSERVATIONS

It is natural to ask what observations of exoplanets are directly connected to atmospheric circulation. At present, these are thermal phase curves of tidally locked hot exoplanets. The best example we have is shown in Figure 10.4, *Spitzer Space Telescope* 8 μm photometry of HD 189733b taken over 30 hours—half of the planet's 2.5-day-period orbit. We assume that at 8 μm the thermal variation of the combined planet and star flux is entirely due to the planet. It is fair to say, then, that the variation in Figure 10.4 corresponds to about 20% variation in planet temperature (from a brightness temperature of 1212 to 973 K) [10]. In contrast, other transiting exoplanets appear to have massive day-night temperature contrasts, up to 1000 K or even higher (e.g., [11]). The crude map shown in Figure 10.5 shows one possible map of HD 189733's 8 μm surface temperature. This map is misleading, because there is no real latitudinal information.

A second connection of atmospheric dynamics to observations is related to the study of Earth as an exoplanet. A pale blue dot observed from afar, Earth is actually the most variable object in our solar system at visible wavelengths. This is due to water clouds and their high albedos in contrast to the dark oceans. Despite the apparent variability of clouds, large-scale patterns persist long enough so that we can determine the rotation rate of Earth as viewed from afar. By binning data to the

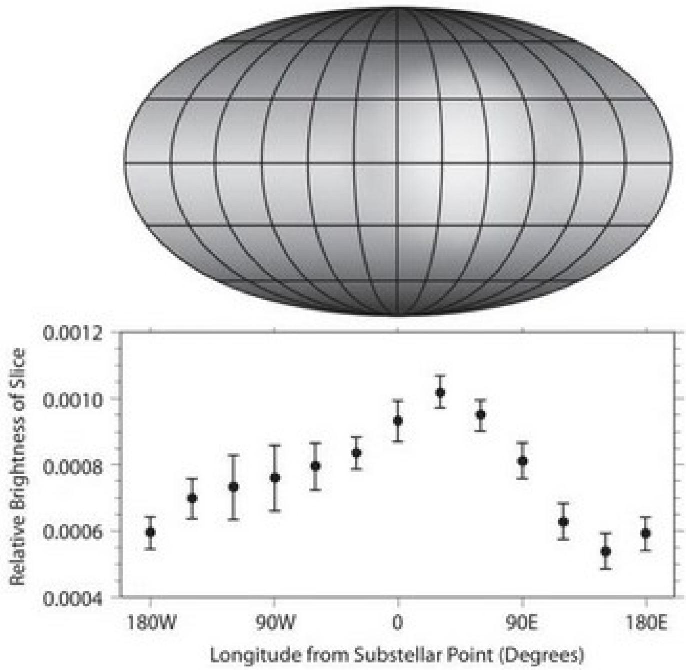

Figure 10.5 Brightness estimates for 12 longitudinal strips on the surface of HD 189733b. a) data are shown as an example a color map. b) data are shown in graphical form. Adapted from [10].

rotation period, we can see variability and attribute the variability to the presence of clouds [12, 13].

One intriguing and promising avenue to understanding what the data can really tell us about exoplanet atmosphere observations may come from inversion of the observed thermal phase functions [14]. This is akin to first trying to understand where hot or cold spots are in the exoplanet atmosphere and next trying to understand the degeneracy in terms of latitude and longitude (and with spectral observations, altitude). Then one could attempt to attribute physical mechanisms for the specific variations. The method could also work for inverting the scattered light phase functions of Earth-like exoplanets, whereby one could infer, very crudely, the presence of continents.

In the future, with the launch of NASA's *James Webb Space Telescope*, spectra as a function of orbital phase will give us a better picture of thermal phase variation. In particular, high signal-to-noise data spectra will not only give us the longitudinally averaged thermal flux but also enable us to understand the flux as a function of altitude (see Chapter 6).

10.6 SUMMARY

Atmospheric circulation is a fundamental topic for planetary atmospheres because it plays such a significant role in how exoplanets look from afar. For many planets, atmospheric circulation acts to minimize horizontal temperature gradients, including equator-to-pole temperature gradients. For solar system planets, the temperature gradients are so small as to not be observable in the hemispherically averaged spectra (except notably in a hemispherically integrated spectrum of Earth centered on Earth's cold poles). For such planets, the 1D average temperature-pressure structure calculations described in Chapter 9 can be used to understand the temperature from observed spectra.

Alternatively, there are exoplanets where atmospheric circulation sets up strong temperature gradients. Evidence comes from thermal phase curve measurements of a few hot Jupiters. For some of these tidally locked planet cores, even the thick mobile atmosphere is not able to circulate the absorbed stellar energy. Instead, hot and cold spots and even hot and cold sides of the planet are maintained. Atmospheric dynamics also plays a role in studies of Earth as an exoplnet—large-scale cloud patterns and their variability may be detectable in the hemispherically integrated signal of visible-wavelength scattered light.

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For further reading

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Two fundamental references on atmospheric circulation:

- Vallis, G. K. 2006. *Atmospheric and Oceanic Fluid Dynamics* (Cambridge: Cambridge University Press).
- Holton, J. R. 1992. *An Introduction to Dynamic Meteorology* (3rd ed; San Diego: Academic Press).

The most thorough reference focused on exoplanet atmospheric circulation is:

- Showman, A. P., Cho, J. Y.-K., and Menou, K. 2010. "Atmospheric Circulation of Exoplanets," in *EXOPLANETS*, ed. S. Seager (Tucson: University of Arizona Press)

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EXERCISES

1. Derive the radiative timescale in equation [10.1].

2. How many bands should Earth have, using equation [10.21]?

3. Derive equation [10.13] from equation [10.12]. Use the definition of $D\mathbf{U}/Dt$ and explain and use the vector identity

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{R}) = -\Omega^2 \mathbf{R}. \tag{10.44}$$

   Here $\boldsymbol{\Omega}$ is taken to be a constant and $\mathbf{R}$ is a vector perpendicular to the axis of rotation. The magnitude of $\mathbf{R}$ is equal to the distance to the axis of rotation.

4. Show that the relation between the total rate of change of a variable is related to the local derivative at a fixed point is

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{D\mathbf{A}}{Dt} - \mathbf{U} \cdot \nabla \mathbf{A}, \tag{10.45}$$

   where $\mathbf{U}$ is velocity.

5. A longitudinally averaged thermal flux of HD 189733b at 8μm is shown in the bottom panel of Figure 10.5, and a map is shown in the top panel. Describe why the map is not unique. Sketch two other representations of the map that also satisfy the actual measured data.