

A691, Midterm 1 Practice Questions:

A) Describe 3 physical processes that broaden spectral lines. Which of these tends to result in the most broadening? Which, the least?

B) Sketch and describe the Bohr Model. What are the key elements of this model, and what useful information does it give us?

C) Student A says that the Sun's optical depth is very large near the center and decreases as one goes outwards, since we see light from the Sun's surface layers but not from its interior. Student B writes that photons are created at the Sun's center, so optical depth is smallest there and biggest toward the outer layers (since it takes those photons so long to finally escape from the Sun). Explain what is right and/or wrong with each person's claim, and describe how you would solve this disagreement.

D) Describe why the spectrum of a molecule looks considerably different from that of an atom.

E) Name and describe three fundamental quantities relating to the absorption of EM radiation (i.e., light). List their units and describe or show how the quantities relate to each other.

F) Pick three substantially different wavelengths λ . For each, give an example of a situation with $\tau\lambda \gg 1$ and another with $\tau\lambda \ll 1$. In each case, explain how you know whether $\tau\lambda$ is large or small.

G) Describe how we have used the Planck blackbody function in A691. What are its units? What other fundamental quantity of radiation has these units?

H) Describe the meaning of Luminosity, Specific Intensity, Flux Density, and Flux. What are the units of these quantities? Which of them are intrinsic to an object (i.e. are distance-independent), and which vary depending on who and where they are observed (i.e., are distance-dependent)?

I) The Figure at right shows spectra (F_λ) of three stars as observed from Earth. Based on this plot:

(a) Rank the stars in surface temperature, from highest to lowest. Or if more information is needed, write "cannot rank."

(b) Rank the stars in distance, from nearest to furthest. Or if more information is needed, write "cannot rank."

(c) Rank the stars in order of peak wavelength, from shortest to longest. Or if more information is needed, write "cannot rank."

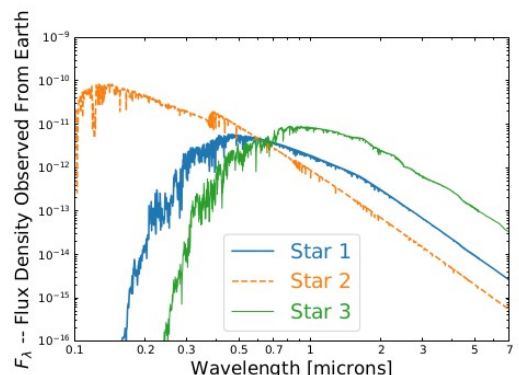


Figure 1: Spectra of three stars as observed from Earth.

J) A star with surface temperature T_* and radius R_* is orbited by a planet with semimajor axis a and radius R_p . Derive an expression for the Flux, F , incident on the planet. What is this quantity called for Earth?

K) A spherical asteroid has radius R_* , surface temperature T_* , and surface brightness equal to that of a blackbody. What is the luminosity of the asteroid? What is its surface Flux Density, F_λ ?

L) Consider a patchy cloud with nonzero $\alpha_\lambda(z)$, whose thickness fluctuates irregularly with altitude up to an altitude of $z = H$; above this the cloud is gone and the sky (and interstellar space beyond) is perfectly clear. The cloud emits no light of its own. A foolish young child is amusing themselves by shining a laser straight up into the cloudy sky.

(a) Show what expression you would need to solve to derive an expression for $\tau_\lambda(z)$, the optical depth of the clouds to this laser beam.

(b) What would be the expression for $I_\lambda(z)/I_{\lambda,0}$, the fraction of the laser light that reaches height z ?

(c) How would one calculate the total optical depth of the cloud, and the fraction of laser light that finally makes it through the cloud and out into space?

M) Sketch Energy-Level Diagrams for (a) the Hydrogen Atom, (b) a diatomic molecule undergoing only rotational transitions; (c) a diatomic molecule undergoing only vibrational transitions. Include at least four energy levels in each diagram.

N) Describe what is meant by “Lyman-alpha”. Give details and be specific.

O) Write the general form of the Equation of Radiative Transfer (as discussed in lecture). Thoroughly explain the meaning of all terms, and the meaning of the overall equation.

P) The general solution to the Equation of Radiative Transfer is $I(\tau) = I_0 e^{-\tau} + S(1 - e^{-\tau})$. Thoroughly explain the meaning of all terms, and the meaning of the overall equation.

Q) Consider a homogeneous cloud deck: it extends all the way to the ground ($z = 0$) and up to an altitude of Z ; above this the cloud is gone and above it the sky is perfectly clear. The cloud has a constant extinction coefficient of α_λ throughout. A distant star’s light, with intensity $I_{\lambda,0}$ at the top of the atmosphere, is coming down toward your observatory. Derive:

a) a general expression for $\tau_\lambda(z)$, for any altitude $z \leq Z$.

b) a general expression for $I_\lambda(z)/I_{\lambda,0}$, the fraction of starlight reaching any altitude $z \leq Z$.

- 1) Imagine a cloud of hydrogen gas (H_2). Let its diameter be ~ 10 parsecs and its number density be $100/\text{cm}^3$. Given that the cross section of H_2 is 0.27 nm^2 , calculate the optical depth through the cloud.
- 2) Define an adiabat. When is it relevant in astronomy?
- 3) Consider a stellar atmosphere in which temperature T , pressure P , and density ρ all decrease with altitude r . Assume a small parcel of gas is perturbed to rise through some small height h , quickly enough that no heat is exchanged while rising. Derive the stability criterion that determines whether or not the parcel and its surroundings are stable against convection.
- 4) Describe an exoplanetary transmission spectrum, and how it differs from a standard emission spectrum such as we more typically observe from galaxies, stars, brown dwarfs, etc.
- 5) Starting with the ideal gas law and equation of hydrostatic equilibrium, derive (i) the dependence of pressure on altitude, $P(z)$, and (ii) the pressure scale height H . Show that on Earth, $H \sim 8 \text{ km}$.
- 6) A planet of radius R_p , and mass M_p orbits a star (radius R^* , mass M^* , temperature T_{eff}) at semimajor axis a . In terms of these quantities, calculate the incident flux F of starlight on the planet. Assuming the planet absorbs all incident starlight and that the planet is in energy balance, calculate the planet's equilibrium temperature. What is the albedo of this planet?
- 7) Given a stellar or planetary density profile $\rho(r)$, calculate (i) the enclosed mass as a function of r , (ii) the internal gravity $g(r)$, (iii) the pressure profile $P(r)$, and (iv) the temperature profile $T(r)$. Sketch all these.
- 8) Consider a stellar or planetary atmosphere in which pressure $P(r)$ increases downward, and local gravity $g(r)$ also points downward. Take a small block of atmosphere with small thickness dr . (i) Draw and write the forces acting on this bit of atmosphere. (ii) Assuming the forces all balance, derive the pressure gradient dP/dr . What is the name of this relation?
- 9) Give a physically motivated explanation as to why limb darkening and limb brightening are observed to occur in stars. Explain the physical conditions needed for either one to occur in a given observation.
- 10) Describe the stellar photosphere, chromosphere, and corona. How do conditions in these regions differ? What effect can they have on observations of spectral lines?
- 11) Clearly describe the difference between brightness temperature, effective temperature, and equilibrium temperature. Give an example of when each one might be relevant.
- 12) Clearly describe the greenhouse effect. In terms of a planet's atmospheric optical depth τ_a , describe what conditions must be met for the greenhouse effect to occur.
- 13) Assume the sun is directly overhead, and is shining down with a flux F_0 , onto a patch of ground with albedo A , surface area S , and surface temperature T_s . (i) What is F_0 , in terms of the star's distance (a), radius (R^*), and temperature (T_{eff})? (ii) How much power is being absorbed by the patch of ground? (iii) How much power is being emitted by the patch of ground? (iv) Assuming energy balance, what is the equilibrium surface temperature, T_s , of the patch of ground?

14) Spectral lines appear partly because atoms and molecules have specific energy transitions, which result in a cross-section as a function of wavelength. (i) Roughly sketch the general shape of σ_λ vs. wavelength λ for some arbitrary spectral line with line center λ_0 . (ii) Explain a plot of τ_λ vs. wavelength will have a similar shape to that of the plot of σ_λ . Explain how the two plots would differ. (iii) Make the simplifying assumption that the number density n and absorption coefficient α_λ are constant with depth throughout the stellar photosphere. Under these approximations, explain why it must be the case that the physical depth we see down into a stellar atmosphere is much less in the line center (at λ_0) than far from the line center (i.e. at λ_1)?

15) For each of the following quantities, state whether the general, overall trend (if any) from center to surface is increasing, decreasing, or indeterminate without further information (and why):

$\rho(r)$, $M_{\text{enc}}(r)$, $g(r)$, $P(r)$, and $T(r)$.

16) In our simplest greenhouse model from lecture, we found that the surface temperature (T_s) of a planet with an atmosphere and with Bond Albedo AB will be increased above T_0 (the temperature with no atmosphere) by a factor

$$\frac{T_s}{T_0} = \left(\frac{1}{2} (2 - \tau_{\text{IR}}) \right)^{-1/4}$$

where τ_{IR} is the atmosphere's infrared optical depth. Explain what ranges of τ_{IR} are valid (and not valid) for this model, and why.

17) Student Y says, "The concept of Energy Balance isn't relevant in stellar atmospheres, because stars create their own energy." Student Z says, "The only cases in which Energy Balance is a useful concept is in planetary surfaces and atmospheres, because planets don't have internal energy sources." Explain what (if anything) is right and/or wrong about these two statements.

18) In class we discussed at length the gray (i.e. wavelength-independent), plane-parallel (i.e. ignoring spherical geometry) stellar atmosphere. Derive the linear limb-darkening law

$$I_0 = a + b \cos \theta$$

under the approximation of a gray, plane-parallel, two-stream atmosphere where $S(\tau) = a + b\tau$.

19) A fairly crude approximation for M, K, and G dwarf stars is that stellar mass, radius and temperature all scale linearly together. Assuming this questionable approximation is valid, plot the stellar luminosity (L_*/L_\odot) of stars with masses from $0.5-1 M_\odot$.