Lecture 14 – Stellar Atmospheres

One of the most important applications of radiative transfer is the description of stellar atmospheres. To begin, we assume that the atmosphere is plane parallel and grey in the sense that the opacity is independent of frequency. Furthermore, we assume that all the energy is carried by radiation with an integrated flux, F. If T_e is the effective temperature of the star and σ_{SB} is the Stephan-Boltzmann constant, then

$$F = \sigma_{SB} T_e^4 \tag{1}$$

In the atmosphere, we write that

$$\mu \frac{dI_\nu}{d\tau} = I_\nu - S_\nu \tag{2}$$

where $\mu = \cos\theta$ is defined relative to the vertical measured outwards. Integrating over all frequencies, this equation becomes:

$$\mu \frac{dI}{d\tau} = I - S \tag{3}$$

Note that locally, we can write that in local thermodynamic equilibrium that:

$$S = \int_0^\infty S_\nu \, d\nu = \int_0^\infty B_\nu \, d\nu = \frac{\sigma_{SB} T^4}{\pi} \tag{4}$$

Previously, we considered the two stream approximation. Here, we solve the equation with the Eddington approximation. We integrate over 4π steradians to find that

$$\frac{d}{d\tau} \int_0^{2\pi} \int_{-1}^{1} I \, \mu \, d\mu d\phi = \int_0^{2\pi} \int_{-1}^{1} I \, d\mu \, d\phi - \int_0^{2\pi} \int_{-1}^{1} S \, d\mu \, d\phi \tag{5}$$

or:

$$\frac{dF}{d\tau} = 4\pi J - 4\pi S \tag{6}$$

where J is the mean intensity. Assuming that the flux is constant through the atmosphere, then we find that:

$$J = S \tag{7}$$

We now multiply the equation of transfer by μ and integrate over 4π steradians to find that:

$$\frac{d}{d\tau} \int_0^{2\pi} \int_{-1}^{1} I \, \mu^2 \, d\mu \, d\phi = \int_0^{2\pi} \int_{-1}^{1} I \, \mu \, d\mu \, d\phi - \int_0^{2\pi} \int_{-1}^{1} S \, \mu \, d\mu \, d\phi \tag{8}$$

We evaluate each of these terms. For the first quantity, we make the diffusion approximation that I is nearly isotropic. In this case, we find that

$$\frac{d}{d\tau} \int_0^{2\pi} \int_{-1}^{1} I \, \mu^2 \, d\mu \, d\phi \approx \frac{dJ}{d\tau} \int_0^{2\pi} \int_{-1}^{1} \mu^2 \, d\mu \, d\phi = \frac{4\pi}{3} \frac{dJ}{d\tau} \tag{9}$$

By definition:

$$F = \int_0^{2\pi} \int_{-1}^{1} I \, \mu \, d\mu \, d\phi \tag{10}$$

Since S is independent of angle, we also have that:

$$\int_0^{2\pi} \int_{-1}^{1} S \, \mu \, d\mu \, d\phi = 0 \tag{11}$$

Using $J = S$, the equation of transfer then reduces to:

$$\frac{4\pi}{3} \frac{dS}{d\tau} = F \tag{12}$$

or:

$$S = \frac{3}{4\pi} F \tau + C \tag{13}$$

where C is a constant. Using our results for S and F given above, we find that

At the outer boundary of the stellar atmosphere where $\tau = 0$, we assume that there is no incoming radiation and therefore:

$$S = \frac{I}{2} = \frac{F}{2\,\pi} \tag{14}$$

Therefore using the boundary condition at $\tau = 0$, then

$$C = \frac{F}{2\pi} \tag{15}$$

or:

$$S = \frac{F}{\pi} \left(\frac{3}{4} \tau + \frac{1}{2} \right) \tag{16}$$

Using the results from above, we therefore find for the temperature that:

$$T^4 = T_e^4 \left(\frac{3}{4} \tau + \frac{1}{2} \right) \tag{17}$$

In an atmosphere with a net flux, we expect a temperature gradient.

The Eddington approximation is very useful as a place to begin to understand a star's atmosphere, but it is obviously incomplete. For example, flux is not exactly conserved through the atmosphere. Another uncertainty is that typically, in a real atmosphere, the opacity varies as a function of frequency. Consider now "real" space with Z pointing outwards along the normal of the atmosphere so that:

$$d\tau_\nu = -\chi_\nu \, \rho \, dz \tag{18}$$