UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2024

Problem Set 1
Due: Friday, 2024/09/06, at the start of class
This problem set is worth **48 points** (+10 possible bonus points).

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **Flux From A Nearby Star [8 pts].** Alpha Centauri A is one star in the triple-star Alpha Centauri system (1.3 parsecs away), and it is a fairly similar star to our Sun (for this problem, assume it is identical to the Sun).
   (a) Roughly how many times weaker is the stellar flux from this star (F_*) that hits the Earth, relative to the Solar flux from the Sun that reaches the Earth ($F_\odot$)? [4 pts]
   (b) Roughly how many times weaker is the stellar flux density from this star ($F_{\nu,*}$) that hits the Earth, relative to the Solar flux density from the Sun that reaches the Earth ($F_{\nu,\odot}$)? [4 pts]

2. **Define Your Terms [10 pts].**
   (a) Explain why the radiation quantity "Intensity" is constant with distance (in empty space), and why this quantity doesn't behave like we're used to when we think about how brightness changes with distance. [5 pts]
   (b) Explain the concept of "solid angle," and how it relates to the more usual angles that we learned about in geometry class. [5 pts]

3. **Radiative Quantities for the Sun and Earth [14 pts].** Consider two astronomical objects: the Earth with $T \approx 300$ K and the Sun with $T \approx 5800$ K:
   (a) Use the Stefan-Boltzmann Law to estimate the surface flux (F, in [W m^{-2}]) of each object. [4 pts]
   (b) Estimate the total luminosity (L, in [W]) of each object. [4 pts]
   (c) Use the Planck Blackbody function to calculate and plot the surface brightness (i.e. the intensity, B_ν) for the two objects on the same axes as a function of wavelength. (Note that all plots you make in this class should have the axes and scales labeled, units specified, and can be either linearly or logarithmically scaled on either axis. You can use your favorite program – Python, GNUPlot, Mathematica, or even a spreadsheet program – to make your plots.) For this plot, your plotting range should extend at least from 0.2 to 20 μm, and your Y-axis will need to be logarithmic to show both spectra at the same time. [6 pts]

4. **Taking it to the Limit [16 pts].** Consider the Planck blackbody function, $B_\nu(T)$.
   (a) Since $e^x \approx 1 + x$ when x is small, show that when studying photons at energies much lower than the Wien peak of the blackbody (or equivalently, when $h\nu \ll kT$), the Planck function reduces to the somewhat simpler form $2\nu^2 kT/c^2$. This is called the "Rayleigh-Jeans Limit," and it is usually applicable at radio wavelengths and often in the infrared. [6 pts]
   (b) Generate the same plot as in the previous plot (B_ν vs. wavelength), but now using the Rayleigh-Jeans approximation instead. (If you want, you can even overplot all curves on the same axes to compare them). [6 pts]
   (c) Discuss the similarities and differences between the two curves that you plotted. [4 pts]

5. **BONUS [10 pts].** Prove the Wien Law: i.e., show that the wavelength $\lambda_{\max}$ for which $B_\nu(T)$ is a maximum is given approximately by $\lambda_{\max} T \approx 3000$ μm K.