UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2024

Problem Set 6
Due: Friday, October 25, 2024, at the start of class (1000 Kansas Time).
This problem set is worth **41 points**.

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **Stellar Structure – of the Earth! I. [17 pts]**. Planets like the Earth are governed by some of the same structure equations that we introduced for stellar interiors. Limit your answers below to two significant figures; the crude model we'll use below often isn't even that accurate.

   (a) Let's make the simplest possible model – assume that the Earth has a constant density, ρ_0. Given the known mass and radius of the Earth, calculate ρ_0. [2 pts]

   (b) Assuming a constant density, calculate and plot the Earth's enclosed mass, $M_{\rm enc}(r)$, from $0 < r < R_\oplus$. What do you calculate for $M_{\rm enc}(r = R_\oplus)$, i.e. for the total mass of the Earth ($M_\oplus$)? Explain how well your answer compares to the true, measured mass of the Earth. [5 pts]

   (c) Assuming a constant density, calculate and plot the Earth's internal gravity field, $g(r)$, for $0 < r < R_\oplus$. Explain how well your answer for $g(r = R_\oplus)$ compares to the true, measured surface gravity at the surface of the Earth. [5 pts]

   (d) Assuming a constant density, calculate and plot the Earth's internal pressure, $P(r)$, for $0 < r < R_\oplus$. Calculate the pressure at the very center of the Earth, $P_c = P(r = 0)$. [5 pts]

2. **Stellar Structure – of the Earth! II. [24 pts]**. For this problem, download the "Preliminary Reference Earth Model" file from `https://crossfield.ku.edu/files/preliminary_reference_earth_model.csv`. For all parts of this problem that involve calculations, be sure to explain your methods and/or show your work. Note that you will need to do some simple numerical integration for this problem: this is best done using your favorite programming language, but spreadsheet programs such as LibreOffice Calc can be made to do this work in a pinch. (Limit numerical answers to three significant figures; I scraped the data file from a figure in a paper, so the data values aren't likely to be more precise than that.)

   (a) Plot the Earth's density profile $\rho(r)$ (from the data file). Explain why you think the density profile has the general shape that it does (don't worry about explaining every little wiggle, but focus on the larger overall features). [4 pts]

   (b) (i) Calculate and plot the Earth's enclosed mass, $M_{\rm enc}(r)$, from $0 < r < R_\oplus$. (ii) Describe and try to explain the general, overall features shown in your plot. (iii) What do you calculate for $M_{\rm enc}(r = R_\oplus)$, i.e. for the total mass of the Earth ($M_\oplus$)? Explain how well your answer compares to the true, measured mass of the Earth. [6 pts]

   (c) Calculate and plot the Earth's internal gravity field, $g(r)$, for $0 < r < R_\oplus$. The plot you get may surprise you – it did me! Describe and try to explain the general, overall features shown in your plot. Explain how well your answer for $g(r = R_\oplus)$ compares to the true, measured surface gravity at the surface of the Earth. [6 pts]

   (d) Calculate and plot the Earth's internal pressure, $P(r)$, for $0 < r < R_\oplus$. Give the central pressure, P_c, for this model and describe how it compares to the central pressure you calculated in the previous problem. [6 pts]