UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2024

Problem Set 6
Due: Wednesday, Nov 13, 2024, before the start of class (by 1000).
This problem set is worth **55 points**.

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **The Gray Plane-Parallel Atmosphere [12 pts]**. In class we discussed at length the gray (i.e. wavelength-independent), plane-parallel (i.e. ignoring spherical geometry) stellar atmosphere.

   (a) Derive the linear limb-darkening law
   $$I_0 = a + b\cos\theta \tag{1}$$
   under the approximation of a gray, plane-parallel, two-stream atmosphere where $S(\tau) = a + b\tau$.

2. **A Model Stellar Atmosphere [30 pts]**. In this problem you will build a simple, 3-layer model of a stellar photosphere.

   (a) Pick three temperatures: T_1 deep in the photosphere (around $\tau \approx 1$, about as deep as we can see), T_2 near the top of the photosphere, and a hotter T_3 up in the chromosphere. Explain how you picked the temperatures that you did. [5 pts]

   (b) Assume the entire stellar photosphere is composed of a material with only a single spectral line (with a Gaussian profile). Pick an optical or infrared central wavelength λ_0 for the line and an effective width σ_λ. Assume the line reaches $\tau = 5$ in the core of the line but $\tau = 0$ everywhere else, at wavelengths far from λ_0. [2 pts]

   (c) Plots! Plot τ_λ over a reasonable range of wavelengths, such that you cover the entire spectral line and some continuum region around the line. Over the same wavelength range, in a separate panel plot blackbody spectra (I_λ or I_ν) for temperatures T_1, T_2, and T_3. [7 pts]

   (d) In terms of T_1 (symbolically, without plugging in specific numbers yet), what is the intensity of radiation I_1 that emerges from the deep, optically thick layer 1? [2 pts]

   (e) Assume that the τ_λ you plotted above corresponds to the optical depth of layer 2 (with T_2). Calculate the intensity I_2 (as a function of wavelength!) that should emerge from the top of layer 2. Show your work. [4 pts]

   (f) Assume that layer 3 (with $T_3 > T_2$) is optically thin, with an optical depth just 10% that of the τ_λ you assumed for layer 2, above. Calculate the intensity I_3 (as a function of wavelength!) that should emerge from the top of layer 3. Show your work. [4 pts]

   (g) Plot the intensities I_1, I_2, and I_3 as a function of wavelength. Which is the emergent intensity that an astronomer would observe? Correctly explain and interpret this spectrum in the context of all we have learned about stellar atmospheres and radiative transfer in this course. [6 pts]

3. **Line Locations vs. Line Widths. [13 pts]** A number of students at times have expressed confusion that although for, e.g., spectral line wavelengths and frequencies $\lambda = c/\nu$ (clearly true!) it is nonetheless *not* the case that the *widths* of these lines are related by $\Delta\lambda = c/\Delta\nu$.

   (a) Pick your favorite spectral absorption line in the UV, optical, or infrared. Write its wavelength λ (in nm) and frequency ν (in Hz). [1 pt]

   (b) Assume that when observed in a particular astronomical object, this absorption line (shown in Fig. 1) has a fractional full-width of 0.2% – i.e. $\gamma = 0.002$ (this "line width" γ has nothing to do with the speed of light).

   Calculate the full line width, $\Delta\nu$, along with the maximum and minimum frequencies of the absorption line profile (marked by the vertical dashed lines in Fig. 1). Don't even think about wavelengths yet! [3 pts].

   (c) Use those the maximum and minimum frequencies to calculate the minimum and maximum wavelength, respectively, of the line profile, as well as the line width in wavelength, $\Delta\lambda$. [3 pts]

   (d) Compare your calculated $\Delta\lambda$ to the result you get by naively calculating $c/\Delta\nu$. Explain why they are not the same, and why the second value would be incorrect. [6 pts]

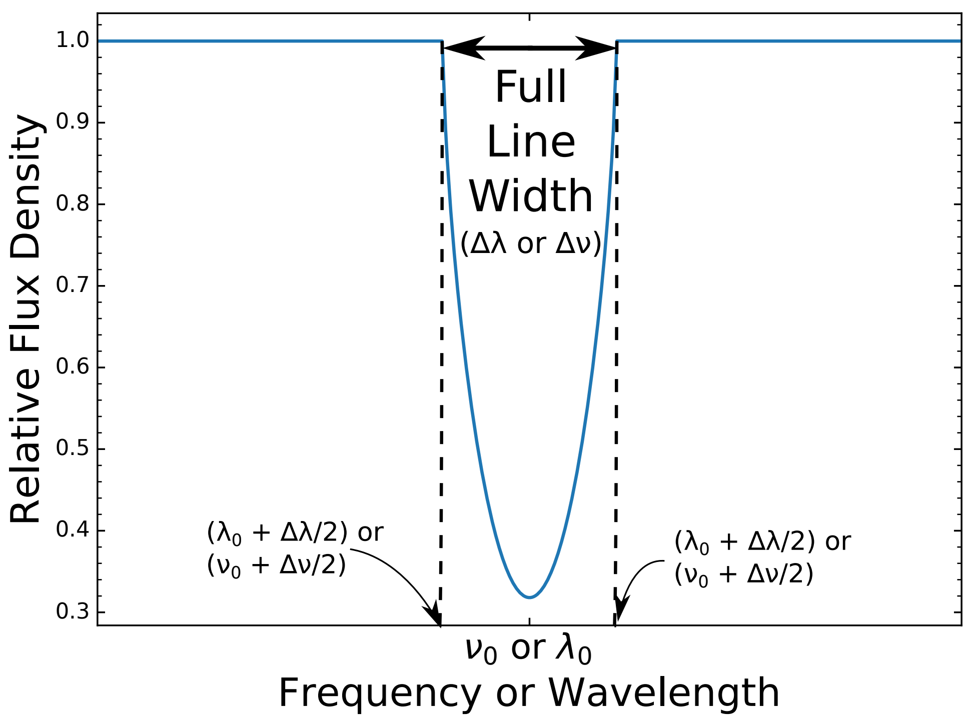

Figure 1: Arbitrary absorption line profile with central frequency and wavelength ν_0 and λ_0 and width $\Delta\nu$ or $\Delta\lambda$.