UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2024

Problem Set 7
Due: Wednesday, Nov 22, 2024, before the start of class (by 1000).
This problem set is worth **48 points**.

As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, type the Pset, and submit a printed copy.
Recommended tools for typesetting your problem set are either LibreOffice or the LaTeX typesetting system available either by download at `https://www.latex-project.org/get/` or in online-only mode via, e.g., `https://www.overleaf.com/`.

1. **Albedos and Temperatures [20 pts]**.

   (a) (5 pts) Assume a planet the size of the Earth, in a 1 AU orbit around a Sun-like star. Plot the planet's expected equilibrium temperature, $T_{\rm eq}$, for the full range of Bond albedos, from $0 \leq A_B \leq 1$.

   (b) (5 pts) Assume the planet above radiates like a simple blackbody (i.e., surface flux given simply by $\pi B_\lambda(T_{\rm eq})$). For the same range of A_B as above, plot the expected wavelength at which the planet's blackbody spectrum would peak.

   (c) (5 pts) Assume a planet the size of Jupiter, around an M dwarf with radius $R_* = R_\odot/2$ and $T_{\rm eff} = 3800$ K. Further assume that the planet has an Earth-like Bond albedo, and plot the planet's expected equilibrium temperature, $T_{\rm eq}$, over a range of orbital separations from 0.05 AU to 5 AU.

   (d) (5 pts) Assume the planet above radiates like a simple blackbody. For the same range of orbital separations as above, plot the expected wavelength at which the planet's blackbody spectrum would peak.

2. **A crude Habitable Zone [14 pts]**

   (a) (5 pts) A fairly crude approximation for M, K, and G dwarf stars is

   $$\frac{M_*}{M_\odot} \approx \frac{R_*}{R_\odot} \approx \frac{T_{\rm eff,*}}{T_{\rm eff,\odot}}. \tag{1}$$

   Assuming this relation holds, plot the stellar luminosity ($L_*/L_\odot$) of stars with masses from 0.5–1 $M_\odot$.

   (b) (4 pts) Again assuming the relation above, calculate the semimajor axis (i.e., orbital separation) for a planet with the same albedo and equilibrium temperature as the Earth but orbiting an M dwarf with $M_* = M_\odot/2$.

   (c) (5 pts) For planets with the same albedo as the Earth, but orbiting different types of stars ($0.5 \leq M_*/M_\odot \leq 1$), plot the orbital separation needed to keep the planets at the same $T_{\rm eq}$ as the Earth.

3. **Brightness Temperatures [14 pts]**

   (a) (5 pts) Calculate the surface flux density of an object emitting as a blackbody, (i.e., $F_{\lambda,\rm surf} = \pi B_\lambda$), at a wavelength of 4.5 μm and with temperatures of 300 K, 1000 K, and 3000 K. Give your answers in units of W/m^2/μm.

   (b) (3 pts) Assume that the objects above are Brown Dwarfs: the size of Jupiter, and 10 pc away. Calculate the observed F_λ for each of the three temperatures above.

   (c) (3 pts) Using JWST, you observe a brown dwarf 10 pc away and measure a flux density of 3×10^{-15} W/m^2/μm. Assuming it is the size of Jupiter, what is its surface flux density?

   (d) (3 pts) What must the surface temperature of this brown dwarf be, in order to explain your observed flux density?