

UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2026

In-Class Problems 4

You may use your OoMA Sheet, your handwritten notes, and a scientific or graphing calculator – and you may work in a group. As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, and write your name.

1. **Equation of Radiative Transfer (absorption only) [5 pts]**. Show that if emission is negligible, that the solution to the equation of radiative transfer is

$$I(\tau) = I_0 e^{-\tau}. \quad (1)$$

Explain what this expression means, in words.

2. **Light-Absorbing Clouds, Part I [14 pts]** In this and the next problem, you will consider how a few types of clouds overhead affect the amount of starlight that reaches a ground-based astronomical observatory. In all cases, assume that the cloud does not emit (or reflect, or scatter) any light. It only absorbs.

First, consider a homogeneous cloud deck: it extends all the way to the ground ($z = 0$) and up to an altitude of $z = H$; above this the cloud is gone and above it the sky is perfectly clear. The cloud has a constant extinction coefficient of α_λ throughout.

A distant star's light, with intensity $I_{\lambda,0}$ at the top of the atmosphere, is coming down toward your observatory.

- (a) How much optical depth has the starlight passed through when it first reaches the top of the cloud, at $z = H$? What fraction of the incident starlight ($I_{\lambda,0}$) reaches altitude $z = H$? [2 pts]
- (b) How much optical depth has the starlight passed through when it reaches your observatory, at $z = 0$? What fraction of the incident starlight reaches altitude $z = 0$? [4 pts]
- (c) Write a general expression for $\tau_\lambda(z)$, for any altitude $z \leq H$. (Make sure that you get the expected behavior for small or large τ !). [4 pts]
- (d) Write a general expression for $I_\lambda(z)/I_{\lambda,0}$, the fraction of starlight reaching any altitude $z \leq H$. [4 pts]