

UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2026

In-Class Problems 12 [21 pts]

You may use your OoMA Sheet, your handwritten notes, and a scientific or graphing calculator – and you may work in a group. As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, and write your name.

Stellar Interiors: II You measure the size and mass of a star in an eclipsing binary system to be R_* and M_* , respectively. What are conditions like in its interior?

1. (a) Assume an unrealistic (but easy-to-calculate) constant density profile, $\rho_r = \rho_0$, which yields

$$M_{\text{enc}}(r) = \frac{4}{3}\pi\rho_0 r^3 \quad (1)$$

and thus

$$g(r) = \frac{4}{3}\pi\rho_0 Gr. \quad (2)$$

→ Starting from the Equation of Hydrostatic Equilibrium, set up the definite integral needed to calculate the pressure profile, $P(r)$, for this simple model [3 pts].

- (b) Calculate $P(r)$. [3 pts]
(c) Sketch $P(r)$, over the range $0 \leq r \leq 2R_*$. [3 pts]
(d) Assume that the star has a uniform composition throughout, with a constant mean molecular weight of μ . Use this and the ideal gas law to calculate the temperature profile, $T(r)$, for this simple model. [3 pts]
(e) Sketch $T(r)$, over the range $0 \leq r \leq 2R_*$. [3 pts]
(f) Calculate the expression for the pressure and temperature at the center of this simplistic star: i.e., $P(r=0)$ and $T(r=0)$. [3 pts]
2. Now assume the same density profile that we saw last time: for $0 \leq r \leq R_*$,

$$\rho(r) = \rho_c \left(1 - \frac{r}{R_*}\right) \quad (3)$$

which yields

$$M_{\text{enc}}(r) = 4\pi\rho_c R_*^3 \left[\frac{1}{3} \left(\frac{r}{R_*}\right)^3 - \frac{1}{4} \left(\frac{r}{R_*}\right)^4 \right] \quad (4)$$

and thus

$$g(r) = 4\pi\rho_c G R_* \left[\frac{1}{3} \left(\frac{r}{R_*}\right) - \frac{1}{4} \left(\frac{r}{R_*}\right)^2 \right]. \quad (5)$$

→ Set up the integral needed to calculate the pressure, $P(r)$, for this more complicated model; you do not need to solve it at this time. You can see why these calculations benefit from the use of a computer, rather than analytical integrals! [3 pts]