UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2026

In-Class Problems 15 [17 pts]
You may use your OoMA Sheet, your handwritten notes, and a scientific or graphing calculator – and you may work in a group. As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, and write your name.

1. Derive the linear limb-darkening law
$$I_0 = a + b\mu \tag{1}$$
under the approximation of a gray, plane-parallel, two-stream atmosphere where $S(\tau) = a + b\tau$ and $\mu \equiv \cos\theta$ [8 pts].

2. Plotting Limb-Darkening: Consider again the humble linear limb-darkening law, $I_0 = a + b\mu$.

   (a) Plot I_0 vs. μ (for $1 \geq \mu \geq 0$), and label the center-of-disk and limb locations, for the following scenarios.
   i. $a = 1$, $b = 0$ (no limb darkening; uniform-disk) [3 pts]
   ii. $a = 0.7$, $b = 0.3$ (weak limb darkening) [3 pts]
   iii. $a = 0.2$, $b = 0.8$ (strong limb darkening) [3 pts]

   (b) Equation 1 is of course only an approximation and does not perfectly match observational data. More precise analyses often use more flexible (though less physically-motivated) limb-darkening "laws" including higher orders of μ.
   More commonly, the linear limb-darkening law is parameterized as $I_0 = 1 - u(1 - \mu)$: with only a single coefficient, such that $I_0(\mu = 1)$ equals unity. Derive an expression for u in terms of a and b [4 pts].

   (c) Plot I_0 vs. μ (for $1 \geq \mu \geq 0$), and label the center-of-disk and limb locations, for:
   i. $I_0 = a + b\mu + c\mu^2$, for $a = 0.3$, $b = 0.93$, and $c = -0.23$ (approximates the Sun at 550 nm) [3 pts]
   ii. $I_0 = 1 - u(1 - \mu)$ for $a = 0.52$ (a variant of the linear form) with $u = 0.52$, overplotted with $I_0 = 1 - a(1 - \mu) - b(1 - \mu)^2$, for $a = 0.40$, $b = 0.24$ [5 pts].