UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2026

In-Class Problems 19 [18+13 pts]
You may use your OoMA Sheet, your handwritten notes, and a scientific or graphing calculator – and you may work in a group. As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, and write your name.

1. **Atmospheric Pressure [18 pts]**. The 1976 US Standard Atmosphere is defined roughly as follows:

z [km]	P [Pa]	T [K]	n [m^{-3}]	ρ [kg m^{-3}]	g [m s^{-2}
0	101300	288			
11	22600	217			
20	5500	217			
32	870	229			
47	110	271			
51	67	271			
71	4.0	215			

   (a) Calculate the temperature gradient, dT/dz, in the lowest region of the atmosphere (i.e., $z < 11$ km). How does your answer compare to the adiabatic lapse rate $(-g/c_P)$ (assuming dry air on Earth has a specific heat capacity of $c_P \approx 10^3$ J kg^{-1} K^{-1})? [3 pts]

   (b) Assuming that the atmosphere has negligible mass compared to the Earth, calculate g in each layer of the atmosphere. Limit your answer to three significant figures. [2 pts]

   (c) Assuming that the atmospheric composition is constant with altitude (average particle weight 29 amu), calculate n and ρ in each layer of the atmosphere. Use an appropriate number of significant figures. How does your answer at sea level compare to the standard density of air of 1.2–1.3 kg m^{-3}? [5 pts]

   (d) Sketch a plot of $\log P$ as a function of z, and sketch T, n, and ρ as a function of P. You'll want T to be on a linear scale, but the other quantitities to be plotted on a logarithmic scale; and, as we've seen in class, the typical convention has the pressure coordinate on the y-axis. [5 pts]

   (e) In class we derived $P(z) = P_0 e^{-z/H}$ under the assumption of constant T and g. In light of the data for this model atmosphere, discuss how reasonable each of these two assumptions really are. [3 pts]

2. **BONUS: Testing A Model With Data [13 pts]**

   (a) Take the $\ln$ (natural log) of both sides of the equation $P(z) = P_0 e^{-z/H}$; show that the result is functionally equivalent to $y = mz + b$, and state what m and b are in terms of relevant atmospheric variables. [4 pts]

   (b) Conduct a simple, unweighted, linear fit to the tabulated data using $\ln P$ and z as above. What do you get for the slope m and offset b? [1 pt]

   (c) Using your model fit, calculate the atmospheric scale height H based on these data. How does your answer compare to the value of H you calculate from $H = kT/\langle m \rangle g$? [4 pts]

   (d) Discuss how good or poor our model $P(z) = P_0 e^{-z/H}$ really does when explaining an atmosphere like the Earth's. [4 pts]