UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Astrophysics I (ASTR 691) — Prof. Crossfield — Fall 2026

In-Class Problems 20 [20+20 pts]

You may use your OoMA Sheet, your handwritten notes, and a scientific or graphing calculator – and you may work in a group *during classtime*. As always, be sure to: show your work, circle or highlight your final answer, list units, use the appropriate number of significant figures, and write your name.

1. **A Slightly Less-Simple Greenhouse [20 pts]**. In class we considered a simple, one-layer atmospheric greenhouse model on a more-or-less Earth-like planet, with IR optical depth $0 < \tau_{IR} \ll 1$ but visible-wavelength optical depth $\tau_{\rm VIS} = 0$; we derived the result

$$T_{\rm surf} = T_{\rm irr} \left(\frac{1-A}{1-\tau_{IR}} \right)^{1/4} \tag{1}$$

where A is the surface albedo, $T_{\rm surf}$ is the surface temperature, and $T_{\rm irr} \equiv T_{\rm eff}(R_*/a)^2$ is the so-called 'irradiation temperature.'

   (a) If $\tau_{IR} = \tau_{\rm vis} = 0$ then the planet has no atmosphere; in this case, show how $T_{\rm irr}$ is related to the equilibrium temperature $T_{\rm eq}$. [2 pts]

   (b) Plot $T_{\rm surf}$ from Eq. 1 as a function of τ_{IR} over the range $0 \leq \tau_{IR} \leq 1$. By what fraction does $T_{\rm surf}$ increase as τ_{IR} increases from 0 to 0.5? [4 pts]

   (c) Explain how Eq. 1 is relevant to the "greenhouse effect" on Earth. [2 pts]

   (d) Show that if τ_{IR} is allowed to take on any value (instead of being restricted to $\ll 1$) that we instead obtain the following relation. [6 pts]

$$T_{\rm surf} = T_{\rm irr} \left(\frac{1-A}{\frac{1}{2}\left(1+e^{-\tau_{IR}}\right)} \right)^{1/4} \tag{2}$$

   (e) Overplot $T_{\rm surf}$ from Eq. 2 on your previous plot, with τ_{IR} over the range $0 \leq \tau_{IR} \leq 2$. In this more accurate model, by what fraction does $T_{\rm surf}$ increase as τ_{IR} increases from 0 to 0.5? by what fraction does it increase as τ_{IR} increases from 0 to infinity? [4 pts]

   (f) What is the atmospheric temperature, $T_{\rm at}$, in terms of $T_{\rm surf}$? [2 pts]

2. **BONUS 1: Building a Better Greenhouse.** Relax the assumption that $\tau_{vis} = 0$, and now allow this quantity to take on any value. Derive a new expression for $T_{\rm surf}$ in terms of $T_{\rm irr}$, A, τ_{IR}, and $\tau_{\rm vis}$. [10 pts]

3. **BONUS 2: Building a Multi-story Greenhouse.** Use the same assumptions as in Eq. 2 above (i.e., negligible visible opacity), but now allow for two layers of atmosphere with temperatures $T_{\rm lo}$ and $T_{\rm hi}$ and with IR optical depths $\tau_{\rm lo}$ and $\tau_{\rm hi}$ (for low and high altitudes, respectively). Derive an expression for $T_{\rm surf}$ in terms of $T_{\rm irr}$, A, τ_{lo}, and τ_{hi}. [10 pts]