UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 792) — Prof. Crossfield — Fall 2021

Problem Set 1 (v1.1, updated Aug 28)
Due: Thursday, Sep 16, 2021, at the start of class.
This problem set is worth **65 points**

1. **Fun With Planets [20 pts]**.

   (a) Use the general transit geometry, Kepler's Laws, and other basic orbital-dynamic relations to show how the bulk density of a star, ρ_*, can be determined solely from the a planet's orbital period (P) and the dimensionless ratio of its semimajor axis and the stellar radius ($a/R*$) — both of which are natural byproducts of the analysis of a transit light curve (with no need for other information directly about the star itself!). In other words, show that

   $$\rho_* \approx \frac{3\pi}{GP^2}\left(\frac{a}{R_*}\right)^3. \tag{1}$$

   If you'd like, you can assume a circular orbit.

   (b) Show that a planet's surface gravity, g_P, can be determined directly from measurements of its star's radial velocity reflex motion (via K, the RV semiamplitude), orbital eccentricity e, the orbital period P, and the transit depth $\delta \approx (R_P/R_*)^2$. In other words, show that

   $$g_P = \frac{2\pi}{P}\frac{(1-e^2)^{1/2}K}{(R_*/a)^2\delta\sin i}. \tag{2}$$

   For this, it will be helpful to know the so-called "mass function" of a spectroscopic binary. This is really just an equality relating the observable and empirical quantities involved, namely

   $$\frac{(1-e^2)^{3/2}K^3P}{2\pi G} = \frac{M_P^3\sin^3 i}{(M_* + M_P)^2} \tag{3}$$

   where M_* and M_P are the stellar and planet mass, and i is the orbital inclination.

2. **Data Analysis [45 pts]** This problem gives walks you through the basic steps of characterizing a transiting exoplanet. You will download two data files for this problem. The first is transit_lightcurve_data.csv, which shows time-series photometry of a star that hosts a transiting planet. The first column of the data file is time (measured in days, from an arbitary starting point) and the second is stellar flux (measured in arbitary brightness units).

   The second file is radial_velocity_data.csv, which shows time-series measurements of the radial velocity of a star that hosts the same planet as described in the previous problem. The first column of the data file is time (measured in days, from the same arbitary starting point) and the second is stellar radial velocity (relative to an arbitrary reference velocity, and measured in meters per second).

   Finally, you can assume that this star is a perfect solar twin – our Sun's Doppelganger, to high precision, in every measureable way.

   (a) Primary Observables: What are the orbital period P, transit depth δ, transit duration (in hours) T_{14}, radial velocity semiamplitude (K) and orbital eccentricity e of this planet? Explain how you calculated each parameter. [20 pts]

   (b) Derived Quantities: What are the planet's orbital semimajor axis a, radius R_P, mass M_P, and the dimensionless ratio a/R_*? Explain how you calculated each parameter. [10 pts]

   (c) Using Eqs. 1 and 2, calculate the planet's surface gravity, g_P, and the stellar density, ρ_* [5 pts].

   (d) Explain what type of planet this is, and how it compares to the planets in our own Solar System. [5 pts]

   (e) Qualitatively describe how all these various system parameters would change if you had made the same observations, but for a system with a star smaller, cooler, and lower-mass than the Sun? [5 pts]