UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 792) — Prof. Crossfield — Fall 2021

Problem Set 2
Due: Tuesday, Oct 26, 2021, at the start of class
This problem set is worth **75 points**

As always, be sure to: show your work, circle your final answer, and show uncertainties and/or the appropriate number of significant figures, where appropriate.

1. **Direct Imaging [15 pts]**.

   5 pts Assume a young, hot, gas giant exoplanet with a temperature of roughly 1000 K. Making reasonable assumptions, calculate and plot the ratio of planet to stellar flux density (F_P/F_*) assuming the planet and its star emit as simple blackbodies and assuming that the planet reflects no light (i.e., that it has an albedo of zero)

   5 pts Assuming reasonable values for all relevant quantities, calculate and plot F_P/F_* for an old, cold gas giant with negligible thermal radiation but with a Jupiter-like albedo of ∼0.3.

   5 pts Calculate the diffraction-limited resolution, λ/D (in arcsec), for a typical modern direct-imaging system. Based on angular-resolution considerations alone, what diameter D would be required to angularly resolve an Earth-Twin system orbiting a Sun-like star at a distance of 10 pc? (Note that this only sets a lower limit on D, since many considerations come into play beyond resolution alone.)

2. **Microlensing [20 pts]**

   4 pts Calculate the Einstein radius, θ_E (in arcsec), for a typical *stellar* microlensing event (neglecting any additional bodies).

   4 pts Calculate θ_E, in arcsec, for a typical *exoplanetary* microlensing event (neglecting any additional bodies, including the planet's own star).

   5 pts Given typical stellar velocities in the Milky Way of order ∼100 km s^{-1}, estimate the typical proper motion (in arcsec/yr) of a typical microlensing lens star.

   7 pts If the true angle between a microlensing source and the unseen lens, β, varies as the proper motion (call it $\dot{\beta}$), then plot the magnification

   $$M(t) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \qquad (1)$$

   as a function of time, where $u \equiv \beta/\theta_E$.

   5 pts Calculate and indicate on your plots above the Einstein crossing time, $t_E = \theta_E/\dot{\beta}$.

3. **Exoplanetary Discovery [15 pts]** Plot, draw, or sketch a diagram showing the parameter space (e.g., M_p or R_p vs. P or a) in which each of the major exoplanetary discovery techniques are most sensitive.

4. **Radiation [25 pts]**

   4 pts A handy rule of thumb is that a star with a V-band (visual) magnitude of 10 corresponds to a photon flux density of roughly 10^4 photons/s/m^2/nm. Explain why this quantity corresponds to a flux density (F_λ) despite the unusual appearance of 'photons' instead of energy units.

   3 pts Assuming that the optical V band extends from roughly 400 to 600 nm, estimate the observed photon flux from such a star.

   3 pts What is the observed photon rate from this star, as observed with a reasonably-sized telescope?

5 pts Neglecting all noise sources but Poisson noise (so S/N $\propto \sqrt{N_{\rm phot}}$), how long would this telescope have to observe such a star to confidently measure a flux difference of 1% (corresponding to a typical transiting hot Jupiter)?

5 pts If the star were $1000\times$ fainter (as in a typical microlensing event), how long would this telescope have to observe it to confidently measure a flux difference of a factor of two (corresponding to a moderate microlensing event)?

5 pts If the original star were instead $1000\times$ brighter (as for some directly-imaged systems), how long would this telescope have to observe to confidently detect the photons from a cold, but highly reflective, exo-gas giant orbiting the star at 2 AU? (Your result will be a lower bound, since in actuality many noise sources are far more punishing that the solely Poisson-limited statistics you are assuming here!).