UNIVERSITY OF KANSAS
Department of Physics and Astronomy
Physical Astronomy (ASTR 792) — Prof. Crossfield — Fall 2021

Problem Set 3
Due: Tuesday, Nov 9, 2021, at the start of class
This problem set is worth **49 points**

As always, be sure to: show your work, circle your final answer, and show uncertainties and/or the appropriate number of significant figures, where appropriate.

1. **Interior Structure of Saturn! [29 pts]**. For this problem, download the file from `https://crossfield.ku.edu/A792_2021B/saturn_interior_nettelmann2013.csv`. This is an interior model of Saturn's density (in g/cc) and temperature (in K) throughout its interior. For all parts of this problem that involve calculations, be sure to explain your methods and/or show your work. Note that you will need to do some simple numerical integration for this problem: this is best done using your favorite programming language, but spreadsheet programs such as LibreOffice Calc can be made to do this work in a pinch. (Limit numerical answers to two significant figures; I scraped the data file from a figure in tje paper, so the data values aren't likely to be more precise than that.)

   (a) Plot Saturn's density profile $\rho(r)$ (from the data file). Explain why you think the density profile has the general shape that it does (don't worry about explaining every little wiggle, but focus on the larger overall features). [4 pts]

   (b) (i) Calculate and plot Saturn's enclosed mass, $M_{\text{enc}}(r)$, from $0 < r < R_{Sat}$. (ii) Describe and try to explain the general, overall features shown in your plot. (iii) What do you calculate for $M_{\text{enc}}(r = R_{Sat})$, i.e. for the total mass of Saturn (M_{Sat})? Explain how well your answer compares to the true, measured mass of Saturn. [6 pts]

   (c) Calculate and plot Saturn's internal gravity field, $g(r)$, for $0 < r < R_{Sat}$. Describe and try to explain the general, overall features shown in your plot. Explain how well your answer for $g(r = R_{Sat})$ compares to the true, measured surface gravity at the surface of Saturn. [6 pts]

   (d) Calculate and plot Saturn's internal pressure, $P(r)$, for $0 < r < R_{Sat}$. Give the central pressure, P_c, for this model and describe how it compares to the central pressure you calculated from the simple order-of-magnitude estimate of $g_{surf} \langle \rho \rangle R_{sat}$. [6 pts]

   (e) Compare your plots (hopefully on a log scale!) of P, T, and ρ. Recall that in an adiabatic environment, $P \propto \rho^\gamma$ and $T \propto P^{(\gamma-1)/\gamma} \propto \rho^{\gamma-1}$. What do you conclude from this comparison about the interior structure of Saturn? [5 pts]

2. **Radiative Transfer [20 pts]**.

   (a) Write the equation of radiative transfer and explain each term in it. [4 pts]

   (b) Assuming a constant source function S_λ and an incoming radiation intensity of $I_{\lambda,0}$, solve for $I_\lambda(\tau_\lambda)$ — i.e., for the radiation intensity as a function of optical depth. [9 pts]

   (c) Calculate the approximate outgoing intensity, I_λ (from the preceding problem) in the following limiting cases: (i) $\tau_\lambda = 0$; (ii) $\tau_\lambda \gg 1$; and (iii) $0 < \tau_\lambda \ll 1$. [9 pts]