UNIVERSITY OF KANSAS
Department of Physics
ASTR 794 — Prof. Crossfield — Spring 2025

Problem Set 5: Temperatures and Albedos
Due: Tuesday, April 22, 2025, in class
This problem set is worth **45 points**.

1. **Equilibrium Temperature [10 pts]** Assume a planet with radius R_P and Bond albedo A_B on a circular orbit with separation a around a star with radius R_* and temperature $T_{\rm eff}$. Assume that the planet's day and night sides emit as blackbodies with temperatures T_D and T_N, respectively, and that the planet's luminosity is driven entirely by absorbed incident radiation.

   (a) Show that
   $$T_D^4 + T_N^4 = \frac{1}{2}\left(\frac{R_*}{a}\right) T_{\rm eff}^4 \left(1 - A_B\right). \tag{1}$$

   (b) For the Earth, plot T_N vs. T_D under this simple model.

2. **Bond Albedos Depend on the Star [20 pts]**

   In this problem, you will investigate how a planet's Bond albedo A_B depends not just on its albedo spectrum A_λ but also on the incident spectrum of the host star (e.g., the spectral type).

   (a) Assume a planet orbiting its star at the "Earth-equivalent insolation distance" (EEID) — this is a crude approximation to the habitable zone that assumes a planet receives the same total incident flux $F_{\rm inc}$ from its star as does the Earth (call this F_{SC}, the Solar Constant flux).

   Derive the EEID in terms of $T_{\rm eff}$, R_*, and F_{SC} [6 pts].

   (b) For main-sequence stars with $M_* \lesssim M_\odot$, a useful (but very approximate!) rule of thumb is that roughly speaking
   $$\frac{R_*}{R_\odot} \approx \frac{T_*}{T_\odot} \approx \frac{M_*}{M_\odot}. \tag{2}$$

   Under this approximation, determine how the EEID scales with stellar radius (or temperature, or mass) [6 pts].

   (c) Assume a trivial but interesting planetary albedo spectrum that is zero for $\lambda \geq 1\,\mu m$ and 0.4 at shorter wavelengths. Assume that all stars radiate as blackbodies, and calculate what fraction of incident starlight is reflected away from this planet if it orbits a star of (i) 1.25 $M_\odot$, (i) 1.0 $M_\odot$, (i) 0.4 $M_\odot$. What is the implied Bond albedo A_B in each case? [8 pts]

3. **Brightness Temperatures [15 pts]**

   (a) (5 pts) Calculate the surface flux density of an object emitting as a blackbody, (i.e., $F_{\lambda,\rm surf} = \pi B_\lambda$), at a wavelength of 4.5 μm and with temperatures of 300 K, 1000 K, and 3000 K. Give your answers in units of W/m^2/μm.

   (b) (3 pts) Assume that the objects above are Brown Dwarfs: the size of Jupiter, and 10 pc away. Calculate the observed F_λ for each of the three temperatures above.

   (c) (3 pts) Using JWST, you observe a brown dwarf 10 pc away and measure a flux density of 3×10^{-15} W/m^2/μm. Assuming it is the size of Jupiter, what is its surface flux density?

   (d) (4 pts) What must the surface temperature of this brown dwarf be, in order to explain your observed flux density?